Hashed Coordinate Sparse Tensor Storage with MATLAB

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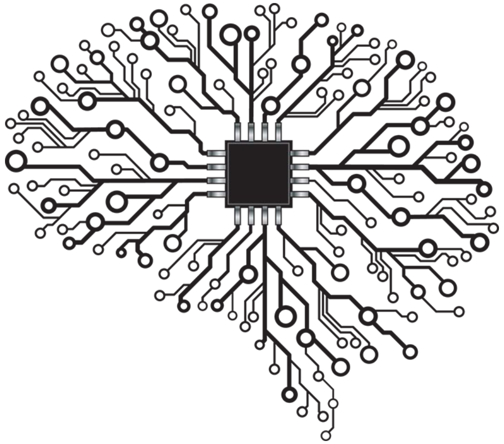
Outline

1. Background and Motivation
2. Existing Storage Formats
3. Hashed Coordinate (HaCOO) format
4. Evaluation
5. Textual Analysis Application
6. Conclusions and Future Goals
7. Q&A

Motivation

- Many important application domains produce and manipulate massive amounts of high-dimensional data
- This data can also be sparse



Source: Laukemann, 2021

Motivation

- For sparse tensors, we want to reduce storage requirements and eliminate meaningless computations on 0 values.

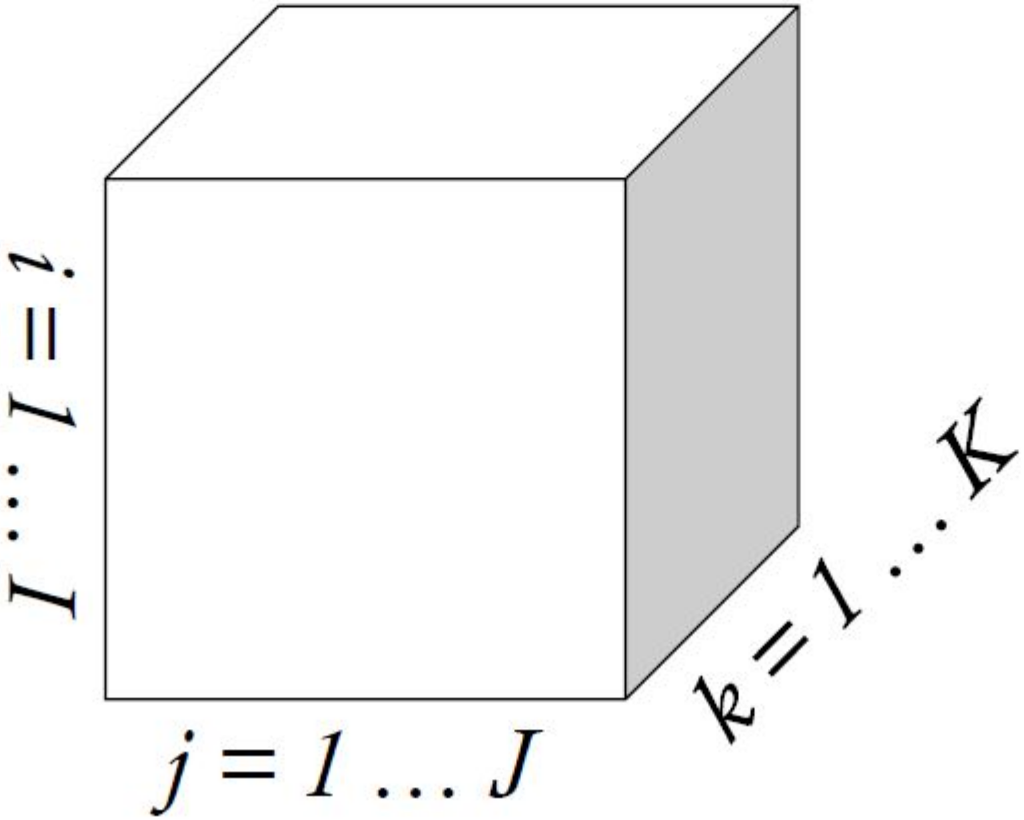
- Challenge: How do we effectively store and compute using high dimensional data?

Background

- A tensor is an *n*-way array

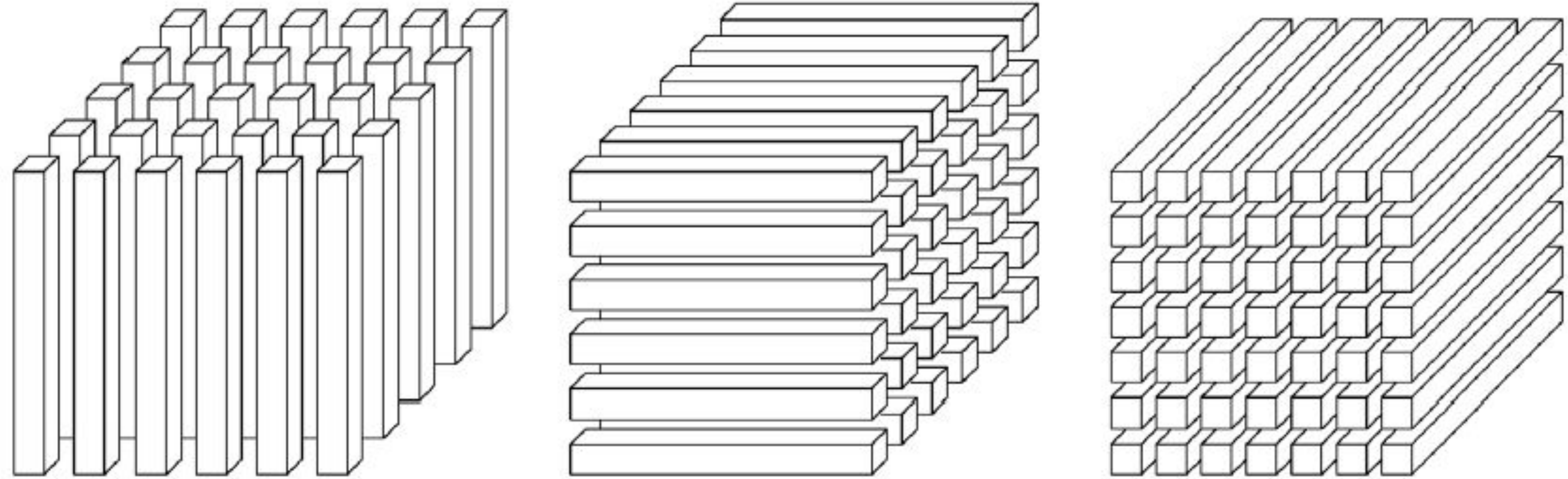
- Each way is referred to as a *mode*

- The number of modes determines a tensor's *order*



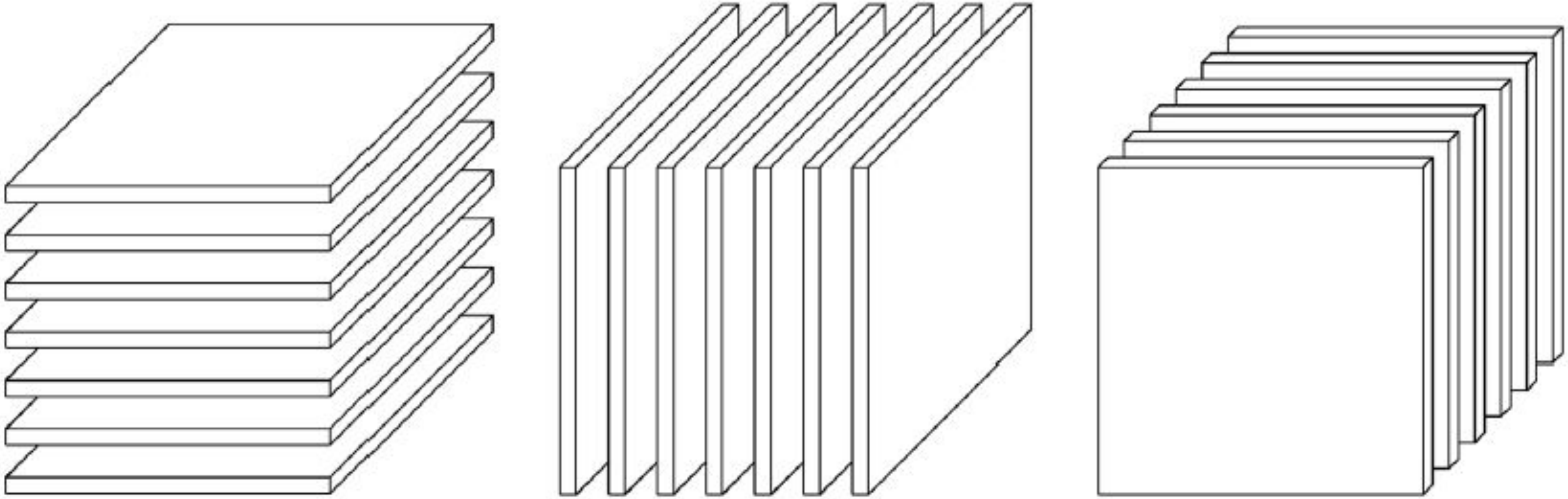
A third-order tensor

Background



(a) Mode-1 (column) fibers: $\mathbf{x}_{:jk}$ (b) Mode-2 (row) fibers: $\mathbf{x}_{i:k}$ (c) Mode-3 (tube) fibers: $\mathbf{x}_{ij:}$

Background



(a) Horizontal slices: $\mathbf{X}_{i::}$ (b) Lateral slices: $\mathbf{X}_{:j:}$ (c) Frontal slices: $\mathbf{X}_{::k}$ (or $\mathbf{X}_k$)

Background

- A tensor can be unfolded, or *matricized* along any of its modes
- $\boldsymbol{\mathcal{X}}_{(n)}$ is a matricized tensor unfolded along the n^{th} mode and is composed of mode-*n* fibers.

Background

Frontal slices:

$$\mathcal{X}_1 = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \quad \mathcal{X}_2 = \begin{bmatrix} 7 & 10 \\ 8 & 11 \\ 9 & 12 \end{bmatrix}$$

$$\mathcal{X}_{(1)} = \begin{bmatrix} 1 & 4 & 7 & 10 \\ 2 & 5 & 8 & 11 \\ 3 & 6 & 9 & 12 \end{bmatrix}, \quad \mathcal{X}_{(2)} = \begin{bmatrix} 1 & 2 & 3 & 7 & 8 & 9 \\ 4 & 5 & 6 & 10 & 11 & 12 \end{bmatrix}$$

Background

$$\mathcal{X}_1 = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \quad \mathcal{X}_2 = \begin{bmatrix} 7 & 10 \\ 8 & 11 \\ 9 & 12 \end{bmatrix}$$

$$\mathcal{X}_{(3)} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 7 & 8 & 9 & 10 & 11 & 12 \end{bmatrix}$$

Background

The *Kronecker product* of matrices $\mathbf{A} \in \mathbb{R}^{I \times J}$ and $\mathbf{B} \in \mathbb{R}^{K \times L}$ is denoted by $\mathbf{A} \otimes \mathbf{B}$, which results in a (IJ) x (KL) matrix:

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{11}\mathbf{B} & a_{12}\mathbf{B} & \dots & a_{1J}\mathbf{B} \\ a_{21}\mathbf{B} & a_{22}\mathbf{B} & \dots & a_{2J}\mathbf{B} \\ \vdots & \vdots & \ddots & \vdots \\ a_{I1}\mathbf{B} & a_{I2}\mathbf{B} & \dots & a_{IJ}\mathbf{B} \end{bmatrix}$$

$$= [\mathbf{a}_1 \otimes \mathbf{b}_1 \quad \mathbf{a}_1 \otimes \mathbf{b}_2 \quad \mathbf{a}_1 \otimes \mathbf{b}_3 \quad \dots \quad a_J \otimes \mathbf{b}_{L-1} \quad a_J \otimes \mathbf{b}_L]$$

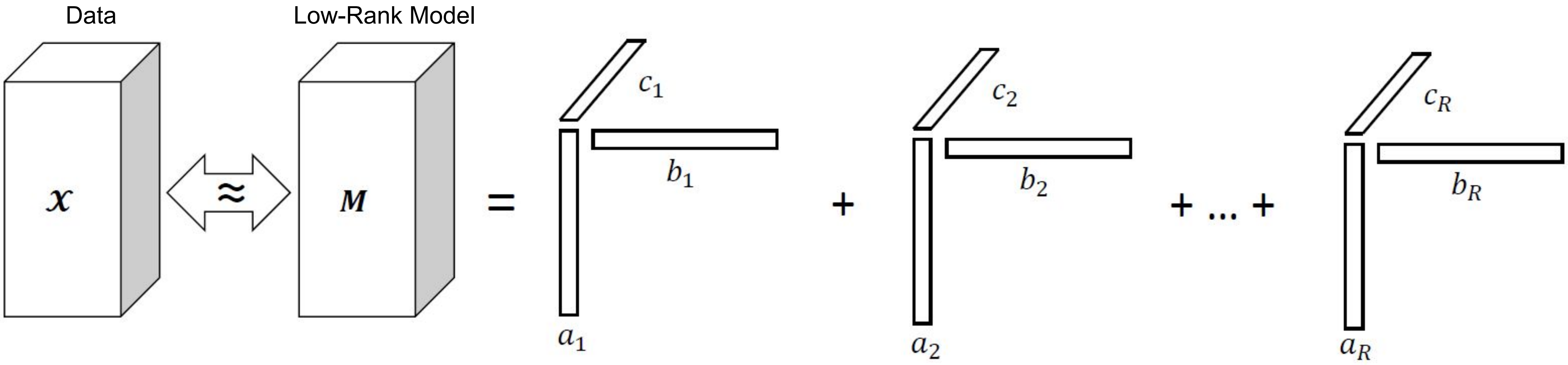
Background

The *Khatri-Rao product* is defined in terms of the Kronecker product. The Khatri-Rao product of matrices $\mathbf{A}^{I \times J}$ and $\mathbf{B}^{M \times J}$ is denoted by $\mathbf{A} \odot \mathbf{B}$, where the resulting matrix is (IM) x J.

$$\mathbf{A} \odot \mathbf{B} = [a_1 \otimes b_1, a_2 \otimes b_2, \ldots, a_n \otimes b_n]$$

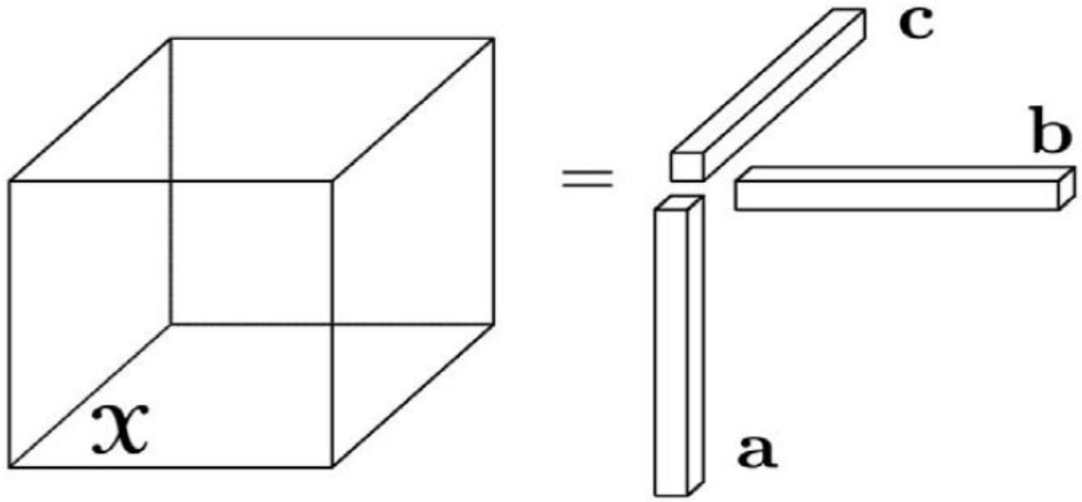
Canonical Polyadic Decomposition (CPD)

- Goal: approximate a mode-*N* tensor X as the sum of *R* rank-1 tensors



Canonical Polyadic Decomposition (CPD)

- An *n*-way tensor is rank-1 if it can be written as the outer product of *n* vectors



Canonical Polyadic Decomposition (CPD)

For the **3-way** case…

- Given 3 vectors:

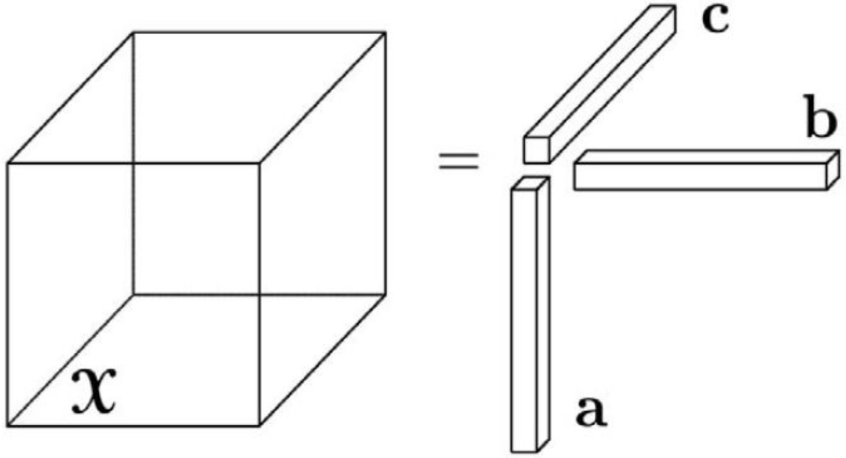
$$\mathbf{a} \in \mathbb{R}^m, \mathbf{b} \in \mathbb{R}^n, \mathbf{c} \in \mathbb{R}^p$$

- Their **outer product** is:

$$\boldsymbol{\mathcal{X}} = \mathbf{a} \circ \mathbf{b} \circ \mathbf{c} \in \mathbb{R}^{m \times n \times p}$$

- Each entry is given by:

$$x(i, j, k) = a(i)b(j)c(k)$$



Canonical Polyadic Decomposition (CPD)

Let $\boldsymbol{\mathcal{X}} \in \mathbb{R}^{I \times J \times K}$ be a three-way tensor.

Compute a CP decomposition with *R* components that best approximates $\boldsymbol{\mathcal{X}}$, i.e., to find:

$$\min_{\hat{\mathcal{X}}} \|\mathcal{X} - \hat{\mathcal{X}}\| \quad \text{with} \quad \hat{\mathcal{X}} = \sum_{r=1}^{R} \mathbf{a}_r \circ \mathbf{b}_r \circ \mathbf{c}_r = \boxed{[\![\mathbf{A}, \mathbf{B}, \mathbf{C}]\!].}$$

Factor matrices

Canonical Polyadic Decomposition (CPD)

Decompose 3-way tensor $\boldsymbol{\mathcal{X}} \in \mathbb{R}^{I \times J \times K}$ into 3 factor matrices:

$$A \in \mathbb{R}^{I \times R}, B \in \mathbb{R}^{J \times R}, C \in \mathbb{R}^{K \times R}$$

The following produces approximate matricizations of the original tensor:

$$\mathbf{X}_{(1)} \approx \mathbf{A}(\mathbf{C} \odot \mathbf{B})^{\mathsf{T}},$$
$$\mathbf{X}_{(2)} \approx \mathbf{B}(\mathbf{C} \odot \mathbf{A})^{\mathsf{T}},$$
$$\mathbf{X}_{(3)} \approx \mathbf{C}(\mathbf{B} \odot \mathbf{A})^{\mathsf{T}}.$$

Canonical Polyadic Decomposition (CPD)

To compute CP, we will use the Alternating Least Squares (ALS) method.

Method:

- Fix matrix **B** and **C** and solve for matrix **A**
- Fix **A** and **C** to solve for **B**
- Fix **A** and **B** to solve for **C**

Canonical Polyadic Decomposition (CPD)

For example:

Matrices **B** and **C** are fixed, solve for **A**.

$$\mathbf{A} = \min_{\mathbf{A}} \| \boldsymbol{\mathcal{X}}_{(1)} - \mathbf{A}(\mathbf{C} \odot \mathbf{B})^T \|_F^2$$

- The optimal solution is given by:

$$\mathbf{A} = \boldsymbol{\mathcal{X}}_{(1)}[(\mathbf{C} \odot \mathbf{B})^T]^\dagger$$

Canonical Polyadic Decomposition (CPD)

- The following form is preferred:

$$\mathbf{A} = \boldsymbol{\mathcal{X}}_{(1)}(\mathbf{C} \odot \mathbf{B})(\mathbf{C}^T\mathbf{C} * \mathbf{B}^T\mathbf{B})^{\dagger}$$

- Only need to calculate the pseudoinverse of an R x R matrix, instead of a (JK) x R matrix

Canonical Polyadic Decomposition (CPD)

CP-ALS:

Matricized Tensor Times Khatri-Rao Product (MTTKRP)

repeat

$$\mathbf{A} = \mathcal{X}_{(1)}(\mathbf{C} \odot \mathbf{B})(\mathbf{C}^T\mathbf{C} * \mathbf{B}^T\mathbf{B})^\dagger$$

$$\mathbf{B} = \mathcal{X}_{(2)}(\mathbf{C} \odot \mathbf{A})(\mathbf{C}^T\mathbf{C} * \mathbf{A}^T\mathbf{A})^\dagger$$

$$\mathbf{C} = \mathcal{X}_{(3)}(\mathbf{B} \odot \mathbf{A})(\mathbf{B}^T\mathbf{B} * \mathbf{A}^T\mathbf{A})^\dagger$$

until maximum number of iterations are reached or error less than ε

end

MTTKRP

Algorithm 1: MTTKRP via Sparse Tensor-Vector products [23]

Input: $indI[M]$, $indJ[M]$, $indK[M]$, $vals[M]$ dense matrices $B^{J \times R}$, $C^{K \times R}$

Output: dense matrix $\mathbf{M}^{I \times R}$

for f=0 *to* F **do**

    for z=0 *to* M **do**

        $t[z] = vals[z] * \mathbf{B}(indJ[z], f) * \mathbf{C}(indK[z], f)$;

    end

    for z=0 *to* M **do**

        $\mathbf{M}(indI[z], f) = \mathbf{M}(indI[z], f) + t[z]$;

    end

end

Source: [11]

Datasets

- Retrieved from FROSTT, Formidable Repository of Open Sparse Tensors and Tools

- Publicly available collection of sparse tensor datasets to facilitate reproducible results

Datasets

- **Uber** tensor: data on Uber pickups in New York City
- **NELL-2** tensor: a smaller version of NELL-1, which is pulled from the Never Ending Language Learner knowledge base, part of a machine learning project from Carnegie Mellon University.
- **Enron** tensor: email data that was publicly released during an investigation by the Federal Energy Regulatory Commission

Datasets

- **Chicago** tensor: crime reports in the city of Chicago
- **Nips** tensor: publications from the NeurIPS Conference on Neural Information Processing Systems.
- **LBNL** tensor: anonymized internal network traffic from Lawrence Berkeley National Laboratory (LBNL).

Datasets

tensor	M	dimensions	storage
uber	3.3M	$183 \times 24 \times 1.1K \times 1.7K$	52.9 MB
nell-2	76.9M	$12.1K \times 9.2K \times 28.8K$	1.51 GB
enron	54.2M	$6K \times 5.7K \times 1.2K$	1.2 GB
chicago	5.3M	$6.2K \times 24 \times 77 \times 32$	80 MB
nips	3.1M	$2.5K \times 2.9K \times 14K \times 17$	58.9 MB
lbnl	1.7M	$1.6K \times 4.2K \times 1.6K \times 4.2K \times 868.1K$	55.1 MB

Sparse Tensor Storage Formats

- Can be roughly grouped into list, block, or tree structures
- Dates range from 2009-2021

Sparse Tensor Storage Formats

- Coordinate (COO) format
  - Kolda and Bader, 2009
  - Stores elements in a list
  - "Standard" sparse tensor storage format
  - Sorted or unsorted

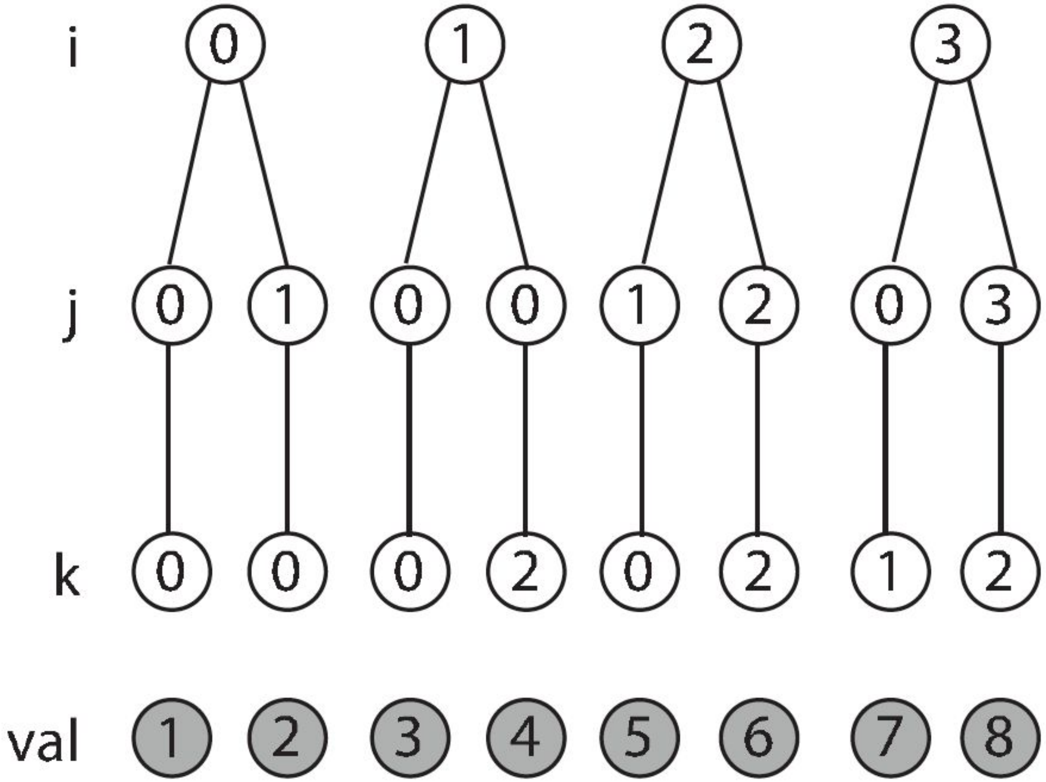
i	j	k	val
0	0	0	1
0	1	0	2
1	0	0	3
1	0	2	4
2	1	0	5
2	2	2	6
3	0	1	7
3	3	2	8

Sparse Tensor Storage Formats

- Compressed Sparse Fiber (CSF) format
  - Smith and Karypis, 2015
  - Tree-based format
  - Mode-specific

SPLATT

(The Surprisingly ParalleL spArse Tensor Toolkit)



Sparse Tensor Storage Formats

- Hierarchical Coordinate (HiCOO) format
  - Li et al., 2018
  - Block-based format
  - Smaller space to search within blocks
  - Smaller indices means less bits to store

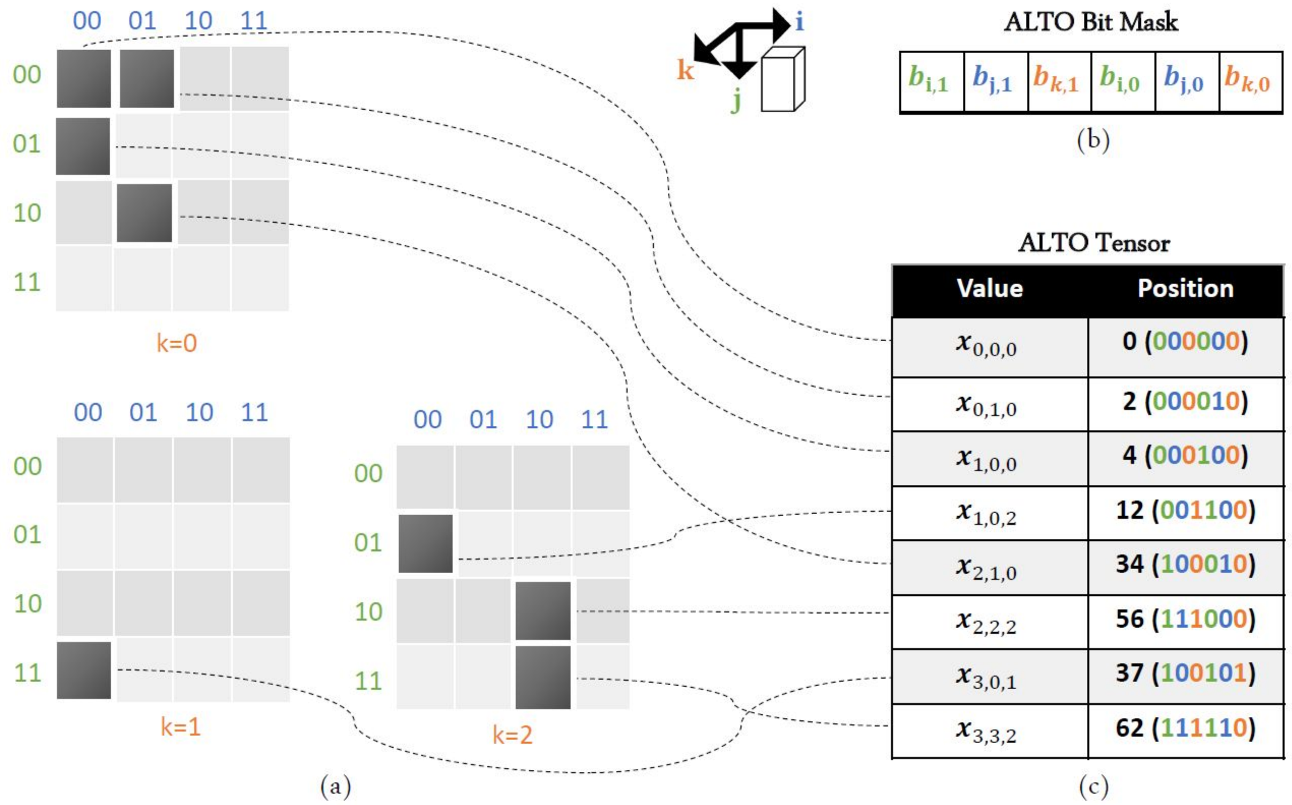
ParTI!
(A Parallel Tensor Infrastructure!)

	bptr	bi	bj	bk	ei	ej	ek	**val**
	0	0	0	0	0	0	0	**1**
B1					0	1	0	**2**
					1	0	0	**3**
B2	3	0	0	1	1	0	2	**4**
B3	4	1	0	0	2	1	0	**5**
					3	0	1	**7**
B4	6	1	1	1	2	2	2	**6**
					3	3	2	**8**

Sparse Tensor Storage Formats

- Adaptive Linearized Storage of Sparse Tensors (ALTO)
  - Helal et. al, 2021
  - Uses bit encoding to order indexes along a line

ALTO library



Why another format?

- Many existing formats:
  - rely on spatial locality of non-zero elements (HiCOO, CSF)
  - do not have a method to insert new tensor values (ALTO, CSF, HiCOO)

Why MATLAB?

- Tensor Toolbox
- Common application that is easily accessible

Hashed Coordinate Format (HaCOO)

- Developed by Robert Lowe, et. al. (2021)

- Separate chaining hash table to store tensor indices and values
  - Uses a low-collision hash function to map indexes into slots within the table
  - Amortized O(1) insertion, update, and retrieval

HaCOO format

i	j	k	val
0	0	0	1
0	1	0	2
1	0	0	3
1	0	2	4
2	1	0	5
2	2	2	6
3	0	1	7
3	3	2	8

COO

Convert each index to Morton code →

morton	val
000000	1
000010	2
100001	3
001010	4
001010	5
111000	6
001101	7
111011	8

Apply Jenkins One-at-a-Time Hash →

morton	val	key
000000	1	1
111000	6	2
		3
		4
		5
		6
		7
111011	8	8
001010 001010	4 5	9
		10
		11
001101	7	12
		13
		14
000010 100001	2 3	15
		16

HaCOO

Hashed Coordinate Format (HaCOO)

- Determine hash values

Algorithm 3: Hash Values

Input: number of buckets in hash table *nbuckets*

Output: sx, sy, sz, $mask$

$bits = \log_2 (nbuckets)$

$sx = \text{ceil}(bits\ /\ 8) - 1$

$sy = 4 * sx\text{-}\ 1$

$sz = \text{ceil}(bits\ /\ 2) - 1$

$mask = nbuckets\text{-}1$

Hashed Coordinate Format (HaCOO)

- Jenkins One-at-a-Time Hash

Algorithm 4: Hash Algorithm

Input: list of non-zero integers $index$

Output: morton value m, hash key k

$m = \text{morton}(index)$

$hash = hash + hash \ll sx$

$hash = hash \oplus hash \gg sy$

$hash = hash + hash \ll sz$

$k = hash \;\&\; mask$

Hashed Coordinate Format (HaCOO)

Example:

idx	morton	step 2	step 3	step 4	k
0, 0, 0	000000	0000000	0000000	000000000	0
0, 1, 0	000010	0000100	0000110	000011110	14
1, 0, 0	000001	0000010	0000011	000001111	15
1, 0, 2	100001	1000010	1100011	111101111	15
2, 1, 0	001010	0010100	0011110	010010110	6
2, 2, 2	111000	1110000	1001000	101101000	8
3, 0, 1	001101	0011010	0010111	001110011	3
3, 3, 2	111011	1110110	1001101	110000001	1

Hashed Coordinate Format (HaCOO)

Tensor	Collision Rate	Mean Probe Depth	Median Probe Depth	Mode Probe Depth	Max Probe Depth
uber	16.43%	1.20	1	1	7
nell-2	26.45%	1.36	1	1	11
enron	37.53%	1.60	1	1	37
chicago	57.27%	25.95	2	1	28
nips	77.31%	4.41	4	4	29
lbnl	89.39%	9.43	1	1	2994

Hashing statistics using the original hash function.

Modified Hashing

- Can the hash function be improved?
- Original algorithm:
  - convert tensor index to Morton code, apply Jenkins hash
- Modified algorithm:
  - concatenate the tensor index, apply Jenkins hash

Modified Hashing

Tensor	Collision Rate	Mean Probe Depth	Median Probe Depth	Mode Probe Depth	Max Probe Depth
uber	17.17%	1.21	1	1	7
nell-2	23.87%	1.31	1	1	9
enron	17.74%	1.22	1	1	8
chicago	26.23%	1.36	1	1	8
nips	16.40%	1.20	1	1	7
lbnl	17.13%	1.21	1	1	7

Results using the modified hash.

HaCOO MATLAB Class

- Create, manipulate, and perform CP decomposition on HaCOO format tensors

- The *htensor* class:
  - hash table, represented as a cell array
  - individual cells contain a matrix of index-value tuples that have hashed into that bucket
  - locations of non-empty buckets
  - modes, number of nonzero elements, maximum chain length, hash parameters, etc.

HaCOO MATLAB Class

- Methods to set/get/search for a tensor index, extract all indexes/values, rehash

- Various class constructors:
  - create a blank HaCOO tensor
  - create a HaCOO tensor from COO tensor or text file

Additional functions to read and write HaCOO tensors from a *.mat* file.

Evaluations

- How do we evaluate HaCOO vs COO in MATLAB?
- Method: simulate “online updates” by inserting elements
- FROSTT tensors are in COO format, pre-sorted and verified to have no duplicate entries

Evaluations

Solution: randomly shuffle the rows of the COO tensor and insert nonzeros one at a time

- Accumulate the time required to insert an element
- Compare wall-clock and CPU time

Building a new COO tensor took days!

- Limit the number of inserted elements to the first *n* entries

Evaluations

Setup:

- Apple MacBook Pro (late 2013)
- 2.4 GHz Dual-Core Intel Core i5 processor
- 128 GB and 4 GB of 1600MHz DDR3L onboard memory

MATLAB functions:

- *tic/toc* to measure elapsed time
- *cputime* to measure total CPU time (summed across threads)

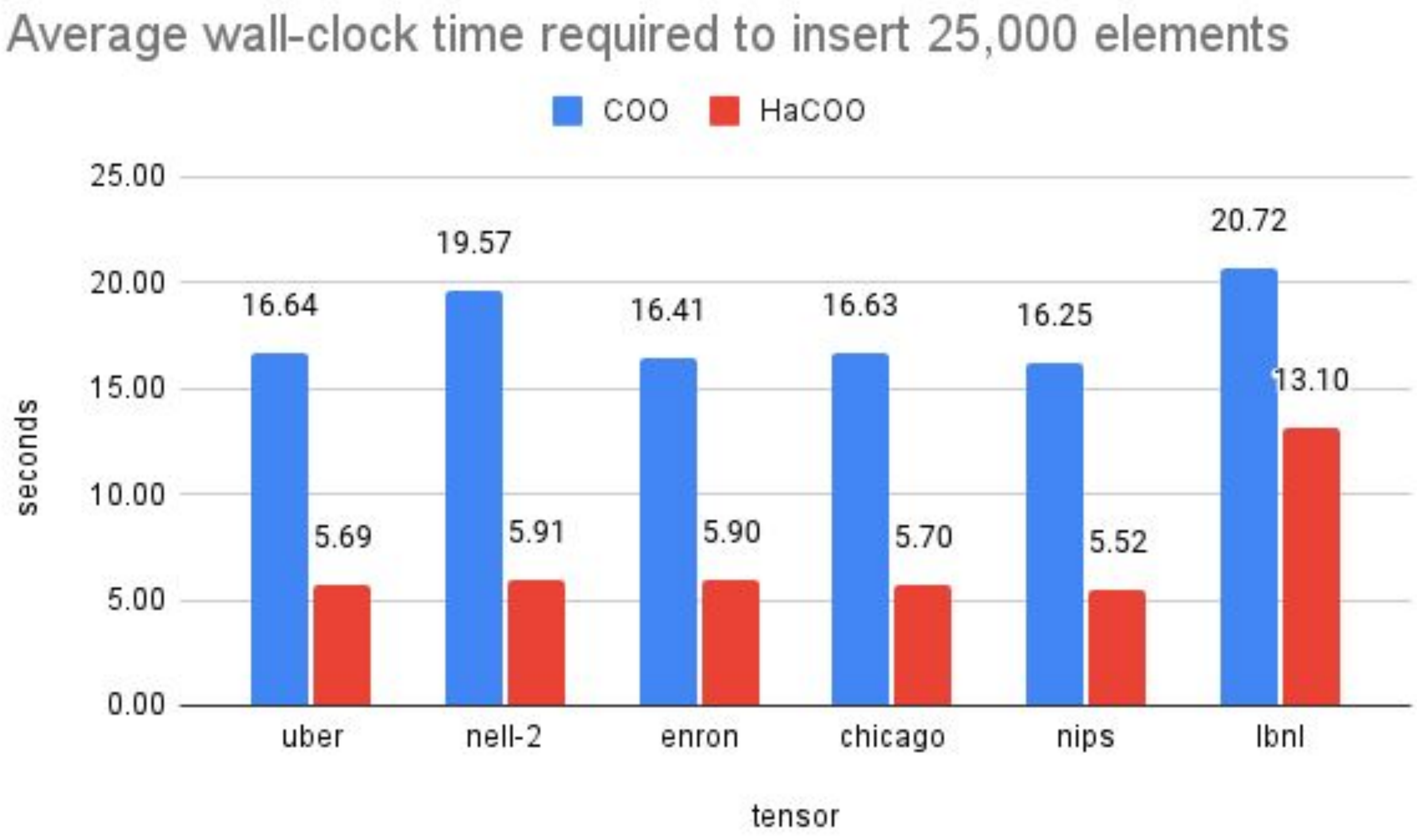
Reported times are averaged over 10 trials

Updating FROSTT Tensors

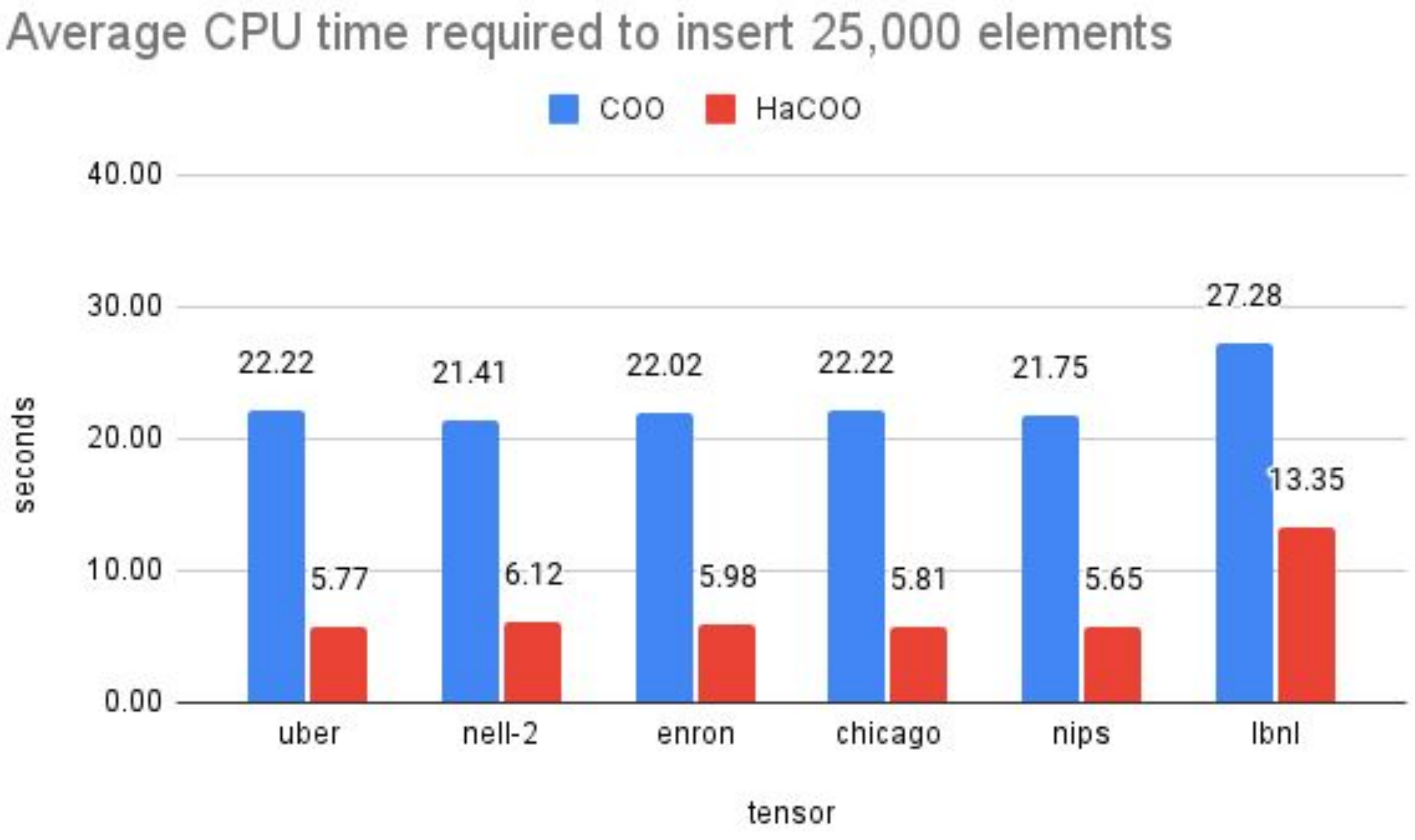
- HaCOO format began to consistently outperform COO once the number of elements inserted reached 25,000.

- Inserting 100,000 random elements from the Uber tensor using HaCOO format yielded around 91-93% reduction in both cumulative wall-clock and CPU time compared to COO format

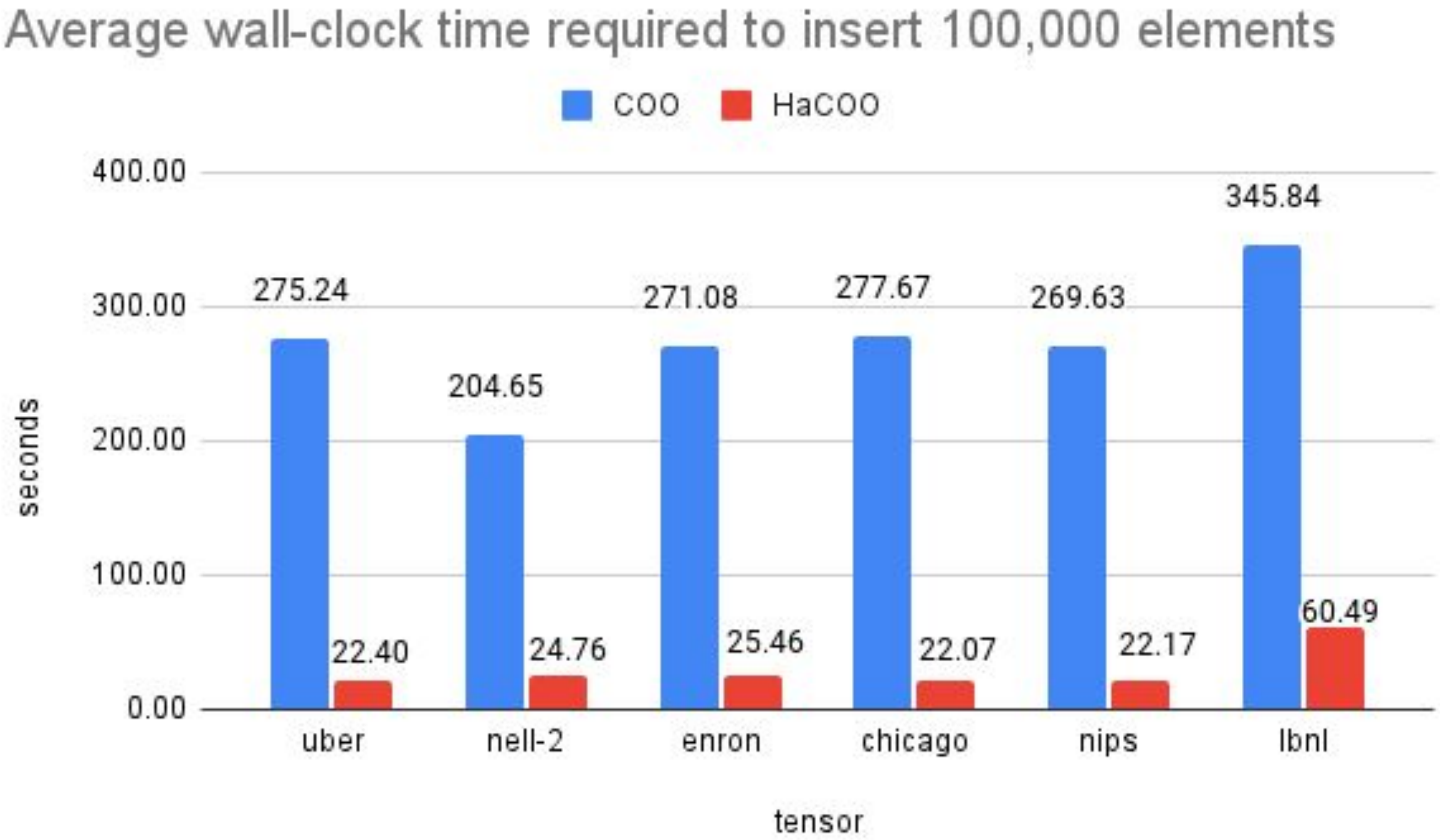
Updating FROSTT Tensors



Updating FROSTT Tensors



Updating FROSTT Tensors



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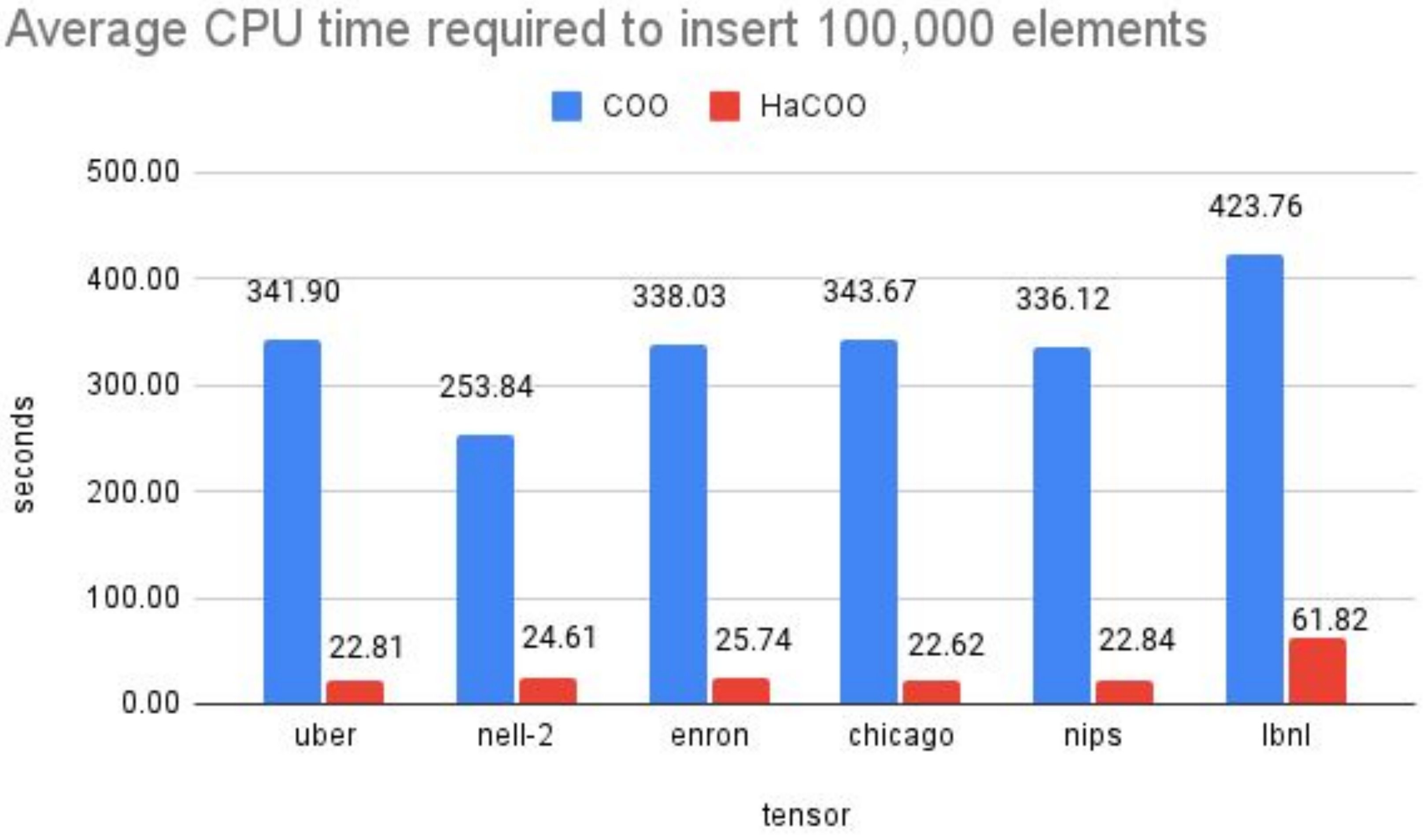
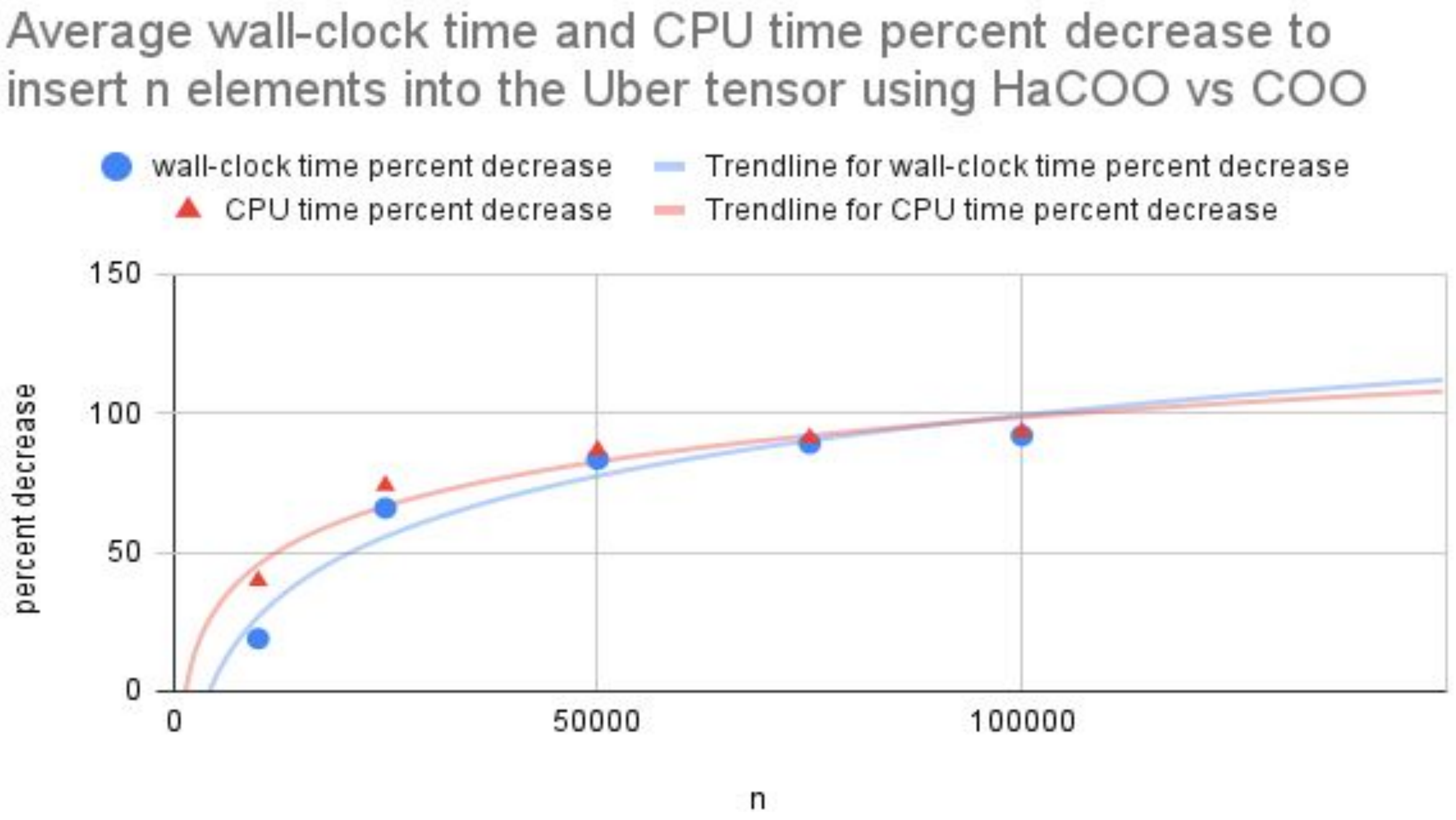
Updating FROSTT Tensors



Evaluations



Evaluations - MTTRKP

- MTTRKP is typically the main bottleneck of CP decomposition

$$A = \mathcal{X}_{(1)}(C \odot B)(C^T C * B^T B)^\dagger$$

- HaCOO method:
  - extract all elements from nonempty cells from the hash table

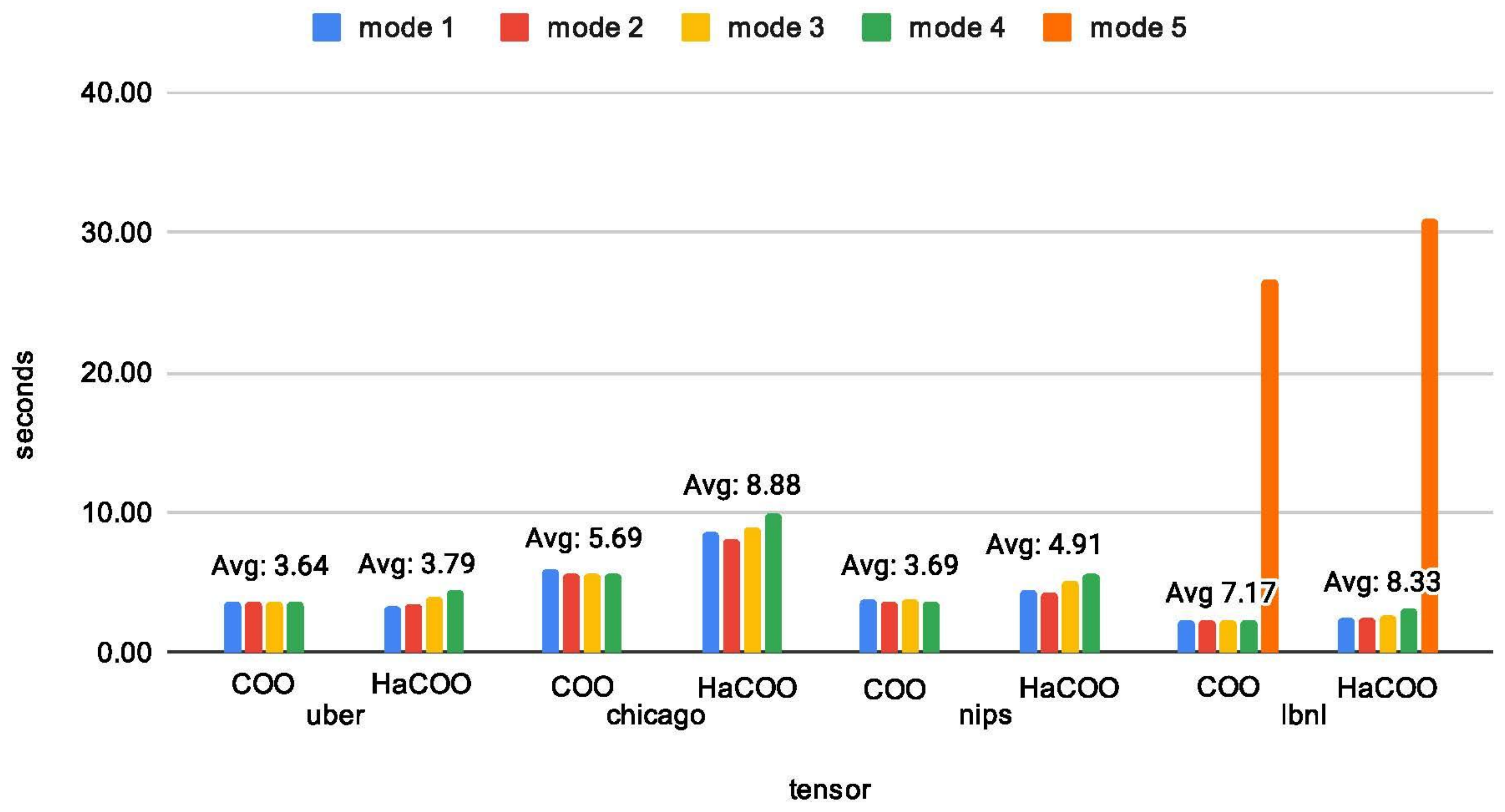
- Time and report averages to compute MTTKRP over every mode

Evaluations - MTTRKP

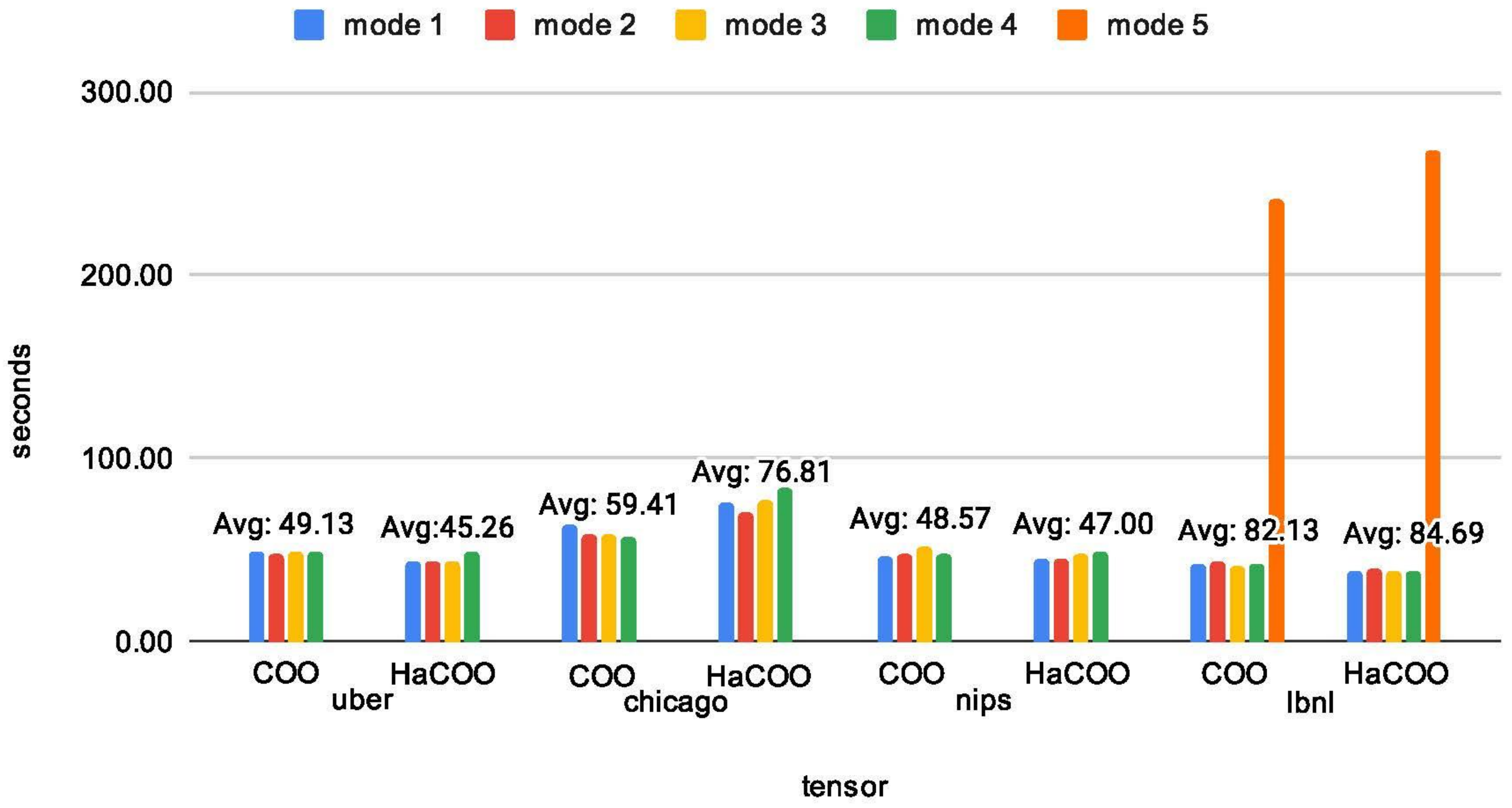
- On average, HaCOO's current MTTRKP method incurs around 26.78% increase in time

- Largest increase observed was over mode 4 of the Chicago tensor
  - 77.88% increase in time to complete
  - Maximum difference in elapsed time over any mode was slightly over 4 seconds

Average wall-clock time required to calculate MTTKRP over all modes



Average CPU time required to calculate MTTKRP over all modes



Text Analysis Application

- Textual Influence model by Lowe (2018)
- Goal:
  - Use sparse tensor decomposition to measure the weight of influence a written document exerts on a target work

First step is to convert all documents into tensors.

Representing Documents as Tensors

Sample document:

The cat jumped on the couch. He yawned and stretched. Then he fell asleep.

Index vocabulary:

1	the	7	yawned
2	cat	8	and
3	jumped	9	stretched
4	on	10	then
5	couch	11	fell
6	he	12	asleep

Representing Documents as Tensors

Counting *n*-grams using a sliding window

Representing Documents as Tensors

The cat jumped on the couch. He yawned and stretched. Then he fell asleep.

1	the	7	yawned
2	cat	8	and
3	jumped	9	stretched
4	on	10	then
5	couch	11	fell
6	he	12	asleep

1, 2, 3	the cat jumped	6, 7, 8	he yawned and
2, 3, 4	cat jumped on	7, 8, 9	yawned and stretched
3, 4, 1	jumped on the	8, 9, 10	and stretched then
4, 1, 5	on the couch	9, 10, 6	stretched then he
1, 5, 6	the couch he	10, 6, 11	then he fell
5, 6, 7	couch he yawned	6, 11, 12	he fell asleep

List of n-grams with corresponding indices

Representing Documents as Tensors

- Unsorted document tensor
- None of the n-grams repeated, so values are 1
- Not all possible *n*-grams will appear, so these tensors are sparse

i	j	k	value
1	2	3	1
2	3	4	1
3	4	1	1
4	1	5	1
1	5	6	1
5	6	7	1
6	7	8	1
7	8	9	1
8	9	10	1
10	6	11	1
6	11	12	1

HaCOO vs COO

- A document tensor's modes grows with the size of the vocabulary
  - COO must spend an increasing amount of time searching if the *n*-gram/index already exists
  - Additional time to do an in-order insert
- HaCOO can spend a constant amount of time to insert

Conference Corpus

5 papers on handwritten digit recognition

2 papers on unrelated topics

45,152 words total

5,236 unique words

Num	Document Information
1	Jessica Lin, Eamonn Keogh, Stefano Lonardi, and Bill Chiu. A symbolic representation of time series, with implications for streaming algorithms. In Proc. DMKD 2003, pages 211. ACM Press, 2003.
2	Andreas Schlapbach and Horst Bunke. Using hmm based recognizers for writer identication and verication. In Proc. FHR 2004, pages 167172. IEEE, 2004.
3	Yusuke Manabe and Basabi Chakraborty. Identity detection from on-line handwriting time series. In Proc. SMCia 2008, pages 365370. IEEE, 2008.
4	Sami Gazzah and Najoua Ben Amara. Arabic handwriting texture analysis for writer identication using the dwt-lifting scheme. In Proc. ICDAR 2007, pages 11331137. IEEE, 2007.
5	Kolda, Tamara Gibson. Multilinear operators for higher-order decompositions. 2006
6	Blei, David M and Ng, Andrew Y and Jordan, Michael I. Latent dirichlet allocation. 2007
7	Serfas, Doug. Dynamic Biometric Recognition of Handwritten Digits Using Symbolic Aggregate Approximation. Proceedings of the ACM Southeast Conference 2017

Shakespeare Corpus

7 works by William Shakespeare

181,760 words total

15,203 unique words

Num	Document Information
1	"Hamlet, Prince of Denmark by William Shakespeare." Project Gutenberg, Nov. 1998, www.gutenberg.org/ebooks/1524. Accessed 10 July 2023.
2	"Julius Caesar by William Shakespeare." Project Gutenberg, Nov. 1998, www.gutenberg.org/ebooks/1522. Accessed 10 July 2023.
3	"Macbeth by William Shakespeare." Project Gutenberg, Nov. 1998, www.gutenberg.org/ebooks/1533. Accessed 10 July 2023.
4	"A Midsummer Night's Dream by William Shakespeare." Project Gutenberg, Nov. 1998, www.gutenberg.org/ebooks/1514. Accessed 10 July 2023.
5	"Othello, the Moor of Venice by William Shakespeare." Project Gutenberg, Nov. 1998, www.gutenberg.org/ebooks/1531. Accessed 10 July 2023.
6	"The Tragedy of Romeo and Juliet by William Shakespeare." Project Gutenberg, Nov. 1997, www.gutenberg.org/ebooks/1112. Accessed 10 July 2023.
7	"Twelfth Night; Or, What You Will by William Shakespeare." Project Gutenberg, Nov. 1998, www.gutenberg.org/ebooks/1526. Accessed 10 July 2023.

Setup

MATLAB scripts to build a vocabulary and build document tensors (Appendix B)

Time how long to build and decompose all document tensors using CP-ALS (50 components)

- constrained and unconstrained vocabularies

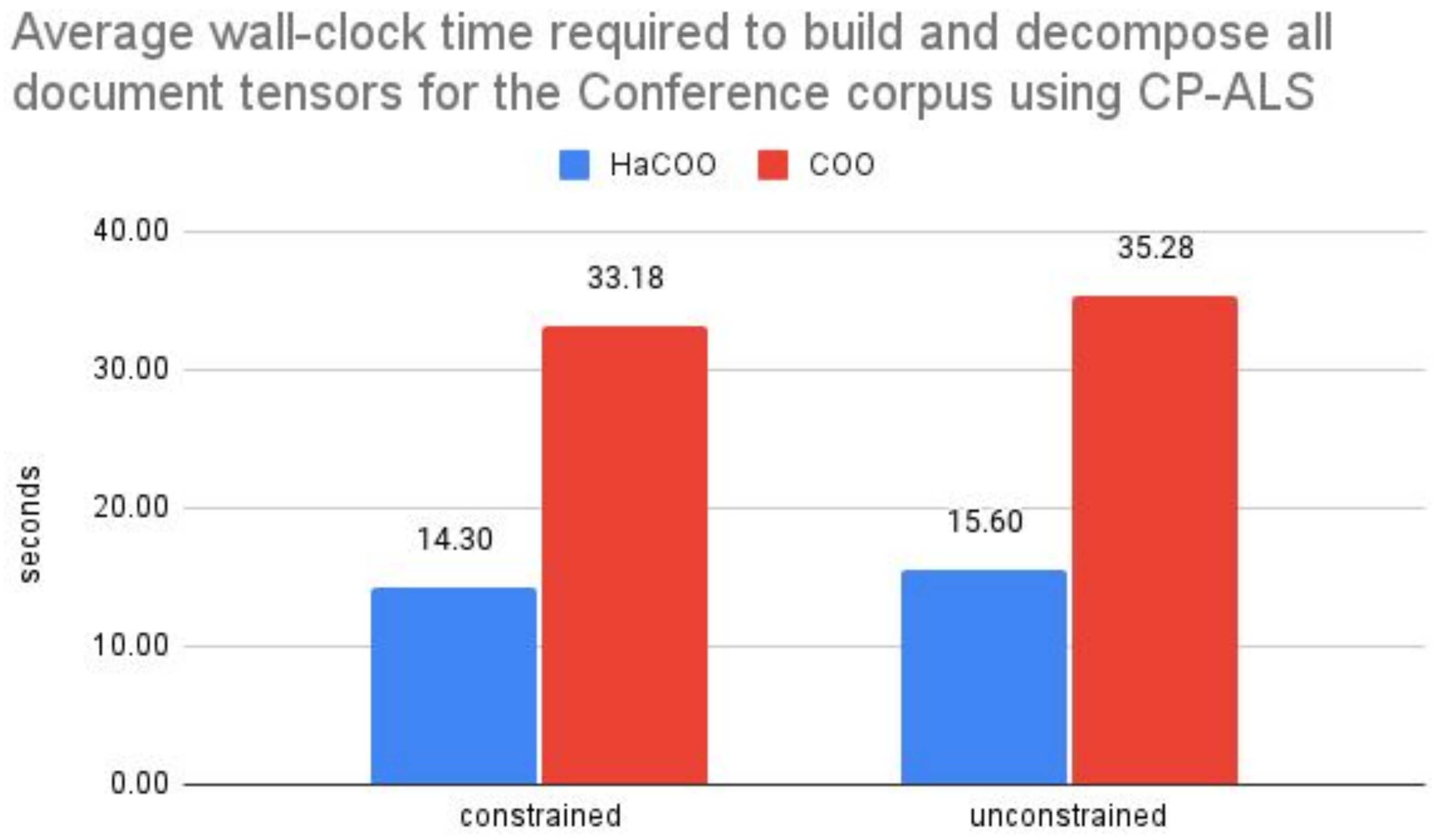
HaCOO: Initial number of buckets was specified to be 1,048,576, or 2^{20}

Results

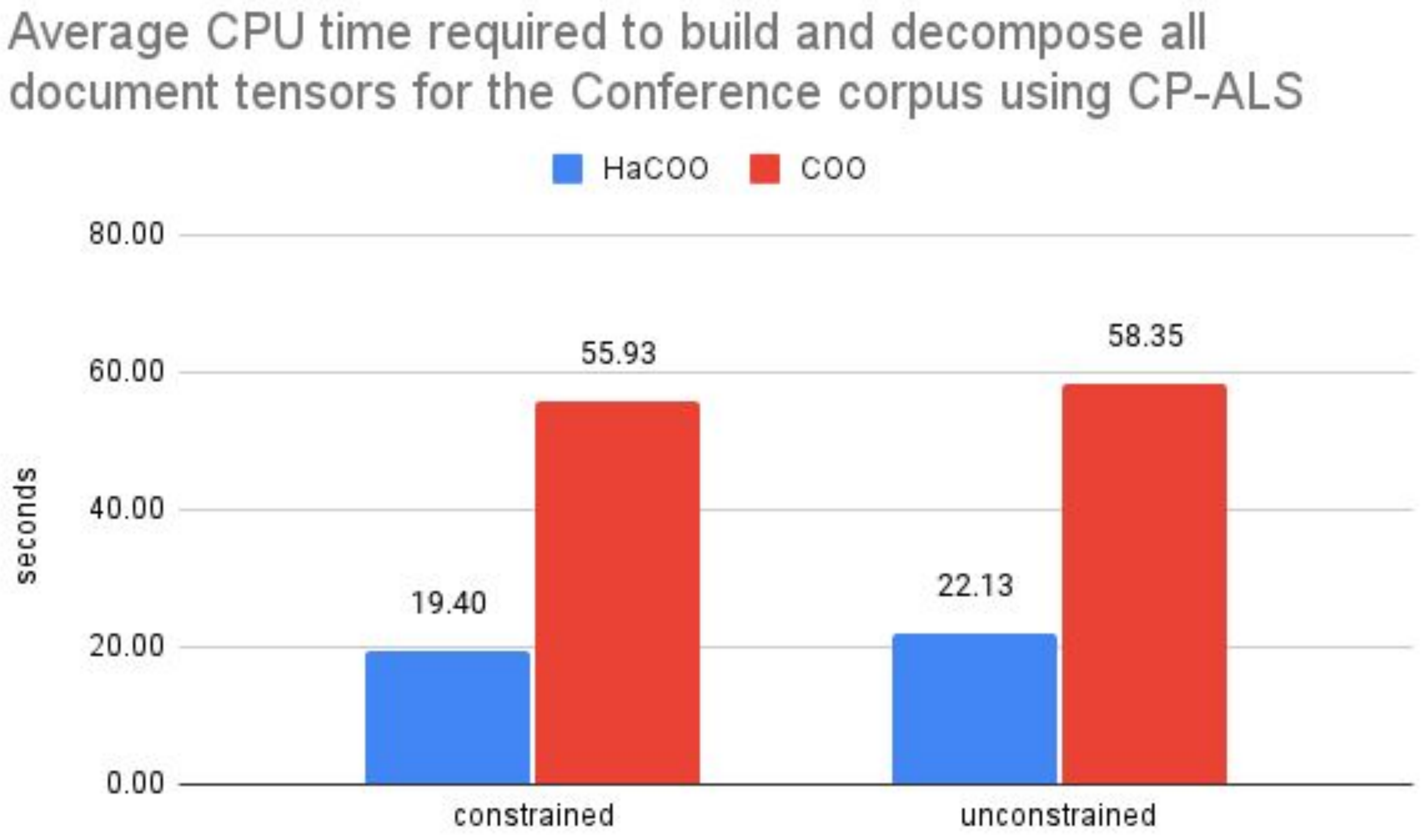
- Conference corpus
  - 44-49% reduction in both wall-clock and CPU time for both the constrained and unconstrained cases

- Shakespeare corpus
  - Constrained:
    - ~14% decrease in wall-clock time
    - ~32% decrease in CPU time
  - Unconstrained:
    - ~72% decrease in wall-clock time
    - ~78% decrease in CPU time

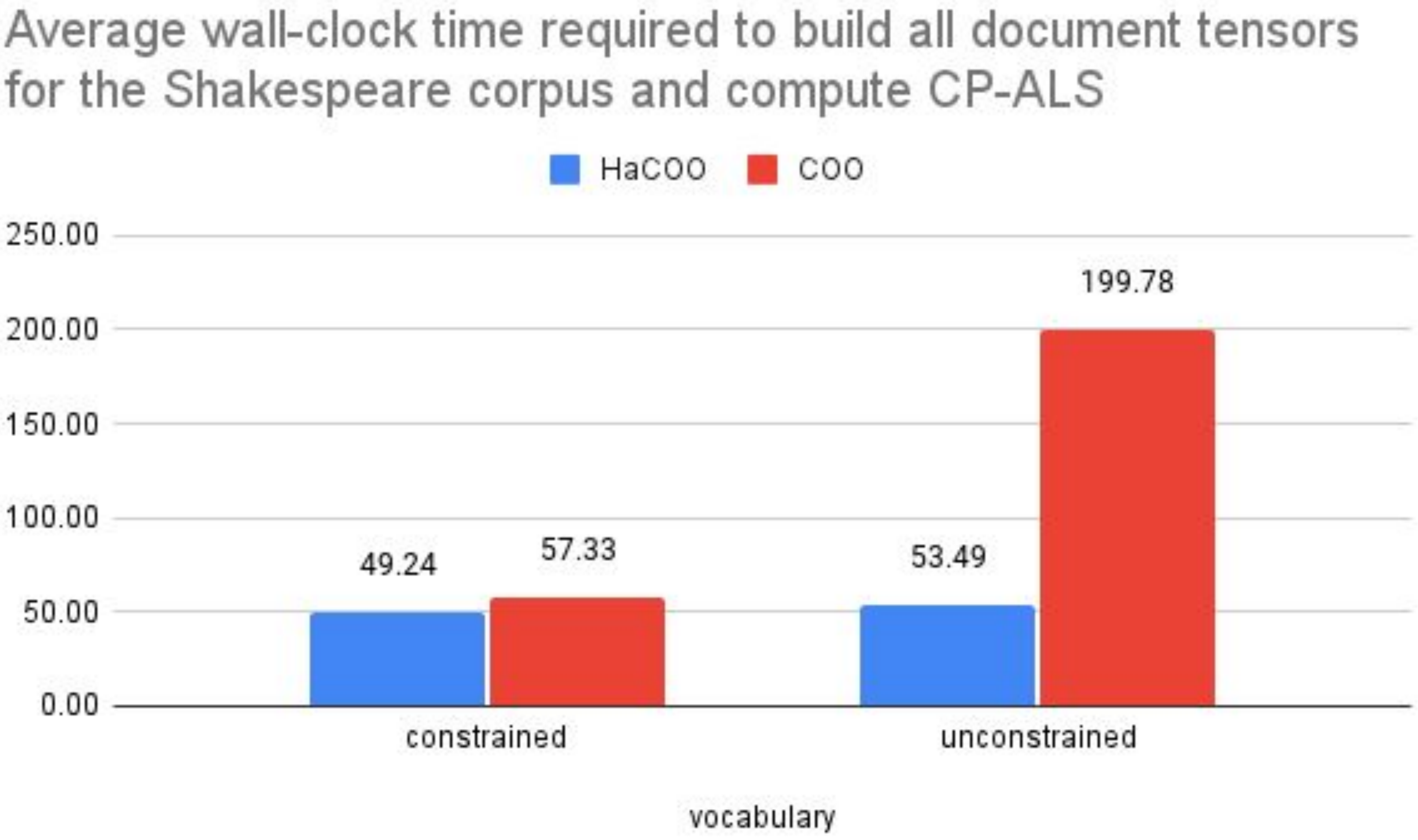
Results - Conference Corpus



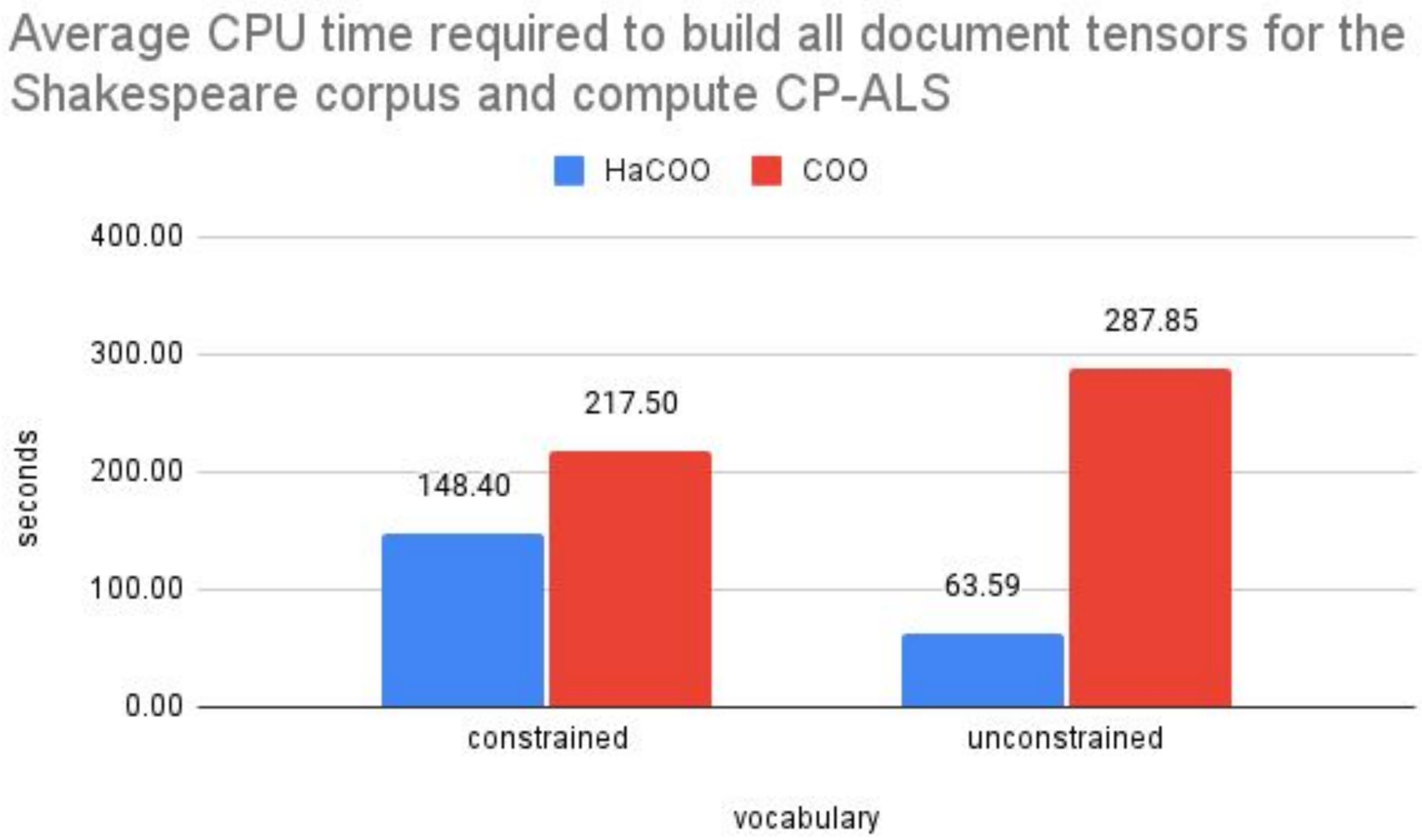
Results - Conference Corpus



Results - Shakespeare Corpus



Results - Shakespeare Corpus



Conclusions

- How to store large, sparse, high-dimensional data?
- Many common sparse tensor storage formats do not allow tensor updates

- HaCOO format benefits:
  - constant time insertion and retrieval
  - MATLAB class to interface with Tensor Toolbox for additional tensor operations without requiring additional hardware or environment setup

Conclusions

- HaCOO outperformed COO format in terms of tensor updates once the number of elements reached a specific threshold.
- CP-ALS was comparable, due to HaCOO's MTTRKP operation incurs a small amount of overhead from extracting tensor elements from the hash table

Future Goals

- MATLAB code clean-up
- What tensor properties contribute to a higher collision rate?
- Further improve hash function
- Workshop on Sparse Tensor Computations
  - University of Illinois Urbana-Champaign
  - October 2023
- Journal article to ACM-TOMS

Q&A

Thank you for your time!

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