Performance Evaluation of the Matlab PCT for Parallel Implementations of Nonnegative Tensor Factorization

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What is the Parallel Computing Toolbox?

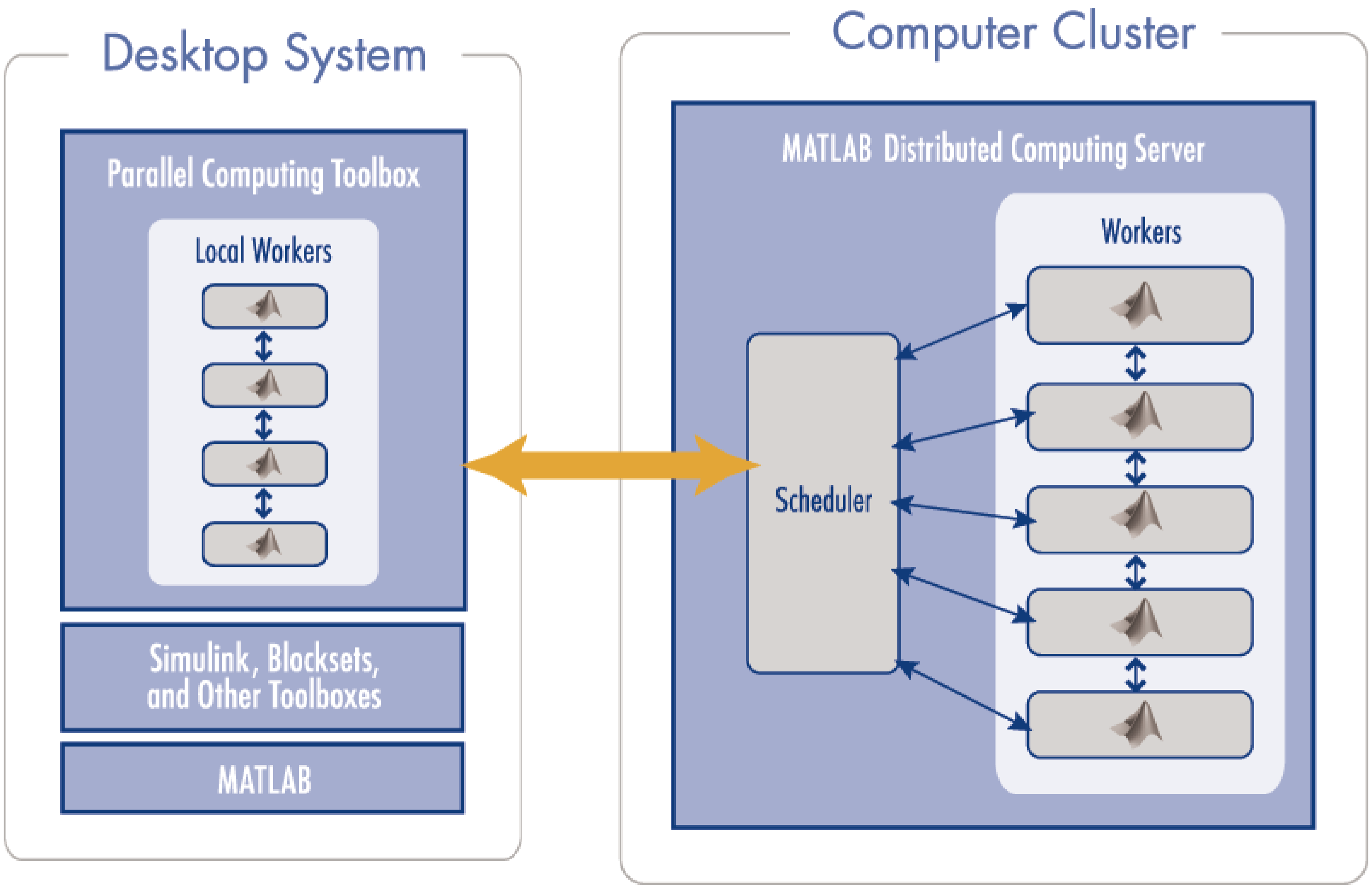
•Lets you solve computationally and data-intensive problems using MATLAB and Simulink on multicore and multiprocessor computers

•Provides support for data-parallel and task-parallel application development

•Provides high-level constructs such as distributed arrays, parallel algorithms, and message-passing functions for processing large data sets on multiple processors

•Can be integrated with MATLAB Distributed Computing Server for cluster-based applications that use any scheduler or any number of workers

Client and Worker nodes


Desktop System
Parallel Computing Toolbox
Local Workers
Simulink, Blocksets, and Other Toolboxes
MATLAB
Computer Cluster
MATLAB Distributed Computing Server
Workers
Scheduler

Application areas of Parallel Computing Toolbox

- Parallel for loops

  Allows individual workers to execute individual loop iterations in parallel

- Offloading work

  Offload work to the worker sessions.

  This is done asynchronously

- Large Data sets

  PCT allows you to distribute that large arrays among the workers, so that each worker has only a part of that array

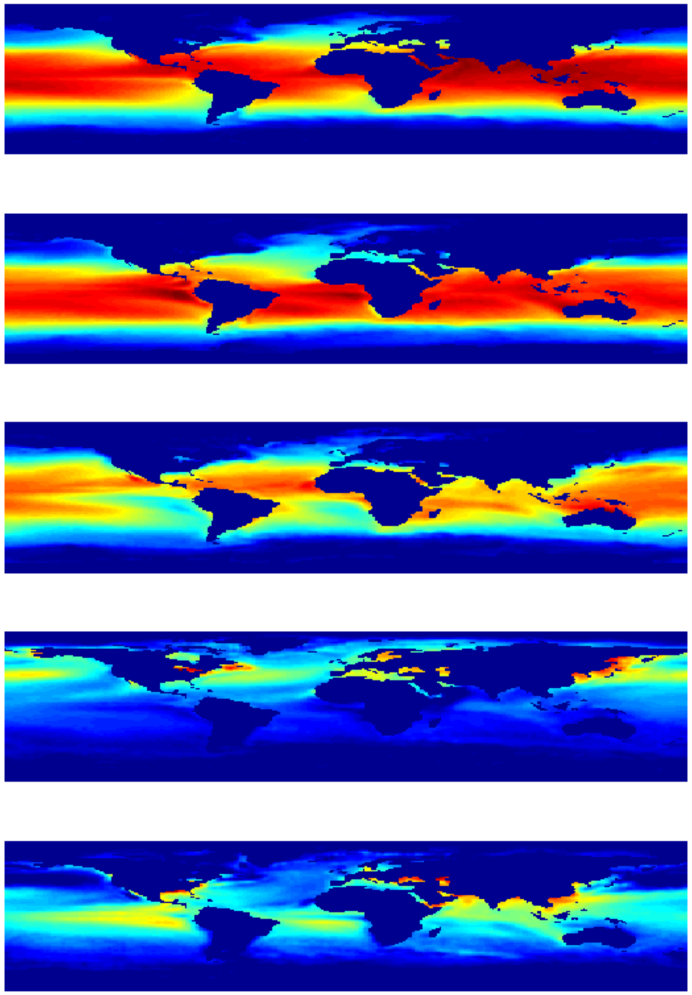
Parallel Computing Toolbox Terminology

Job	consists of all tasks that would perform parallel code execution
Task	Task is tied to the parallel function to execute
Worker / Node	Matlab sessions on other cores/ clusters to which parallel work is offloaded
Client	Matlab session where sequential code is executed and parallel job is setup
Job Manager	Setup on the scheduler and manages job – node allocation
Job queue	Sequence of jobs to be executed by the worker nodes

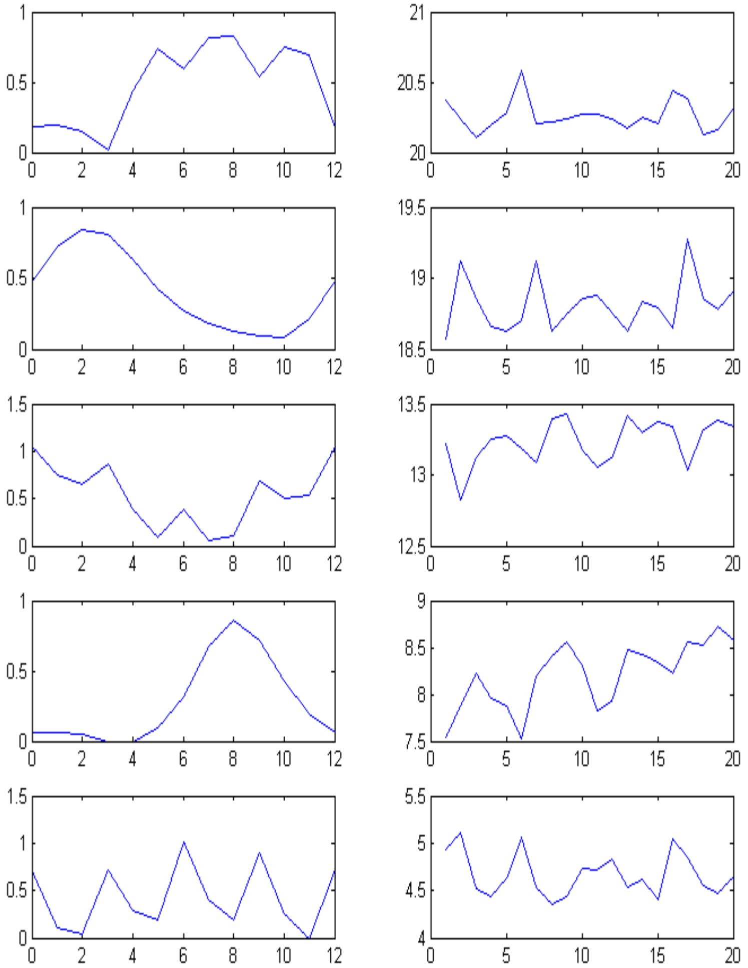
Non Negative Tensor Factorization

- Data mining techniques are commonly used for the discovery of *interesting patterns*
- Study sought to identify regions (or clusters) of the earth which have similar short- or long-term characteristics.
- Earth scientists are particularly interested in patterns that reflect deviations from normal seasonal variations

Patterns from the climate data



Global map of sea surface temperature patterns



Monthly and yearly variations of sea surface temperature patterns

Non Negative Tensor Factorization

- Eigensystem-based analysis driven by principal component analysis (PCA) and the singular value decomposition (SVD) has been used to cluster climate indices
- Orthogonal matrix factors generated by the SVD are difficult to interpret
- Among other data mining techniques, Nonnegative Matrix Factorization (NMF) has attracted much attention
- In NMF, an $m \times n$ (nonnegative) mixed data matrix X *is approximately factored into a product of* two nonnegative rank-k *matrices, with* k *small compared to* m *and* n, $X \approx WH$.
- *W and H can provide a physically* realizable representation of the mixed data *W and H can provide a physically* realizable representation of the mixed data
- Nonnegative Tensor Factorization (NTF) is a natural extension of NMF to higher dimensional data.
- In NTF, high-dimensional data, such as 3D or 4D global climate data, is factored directly and is approximated by a sum of rank-1 nonnegative tensors.

Non Negative Tensor Factorization

- The ALS approach separates the NTF problem into three semi-NMF sub problems within each iteration, i.e.

- Given **X and Y, we solve for Z by**

$$\min_{Z} \phi(Z) = \left\| T_z - (X \bullet Y) Z \right\|_F^2$$

- Given **X and Z, we solve for Y by**

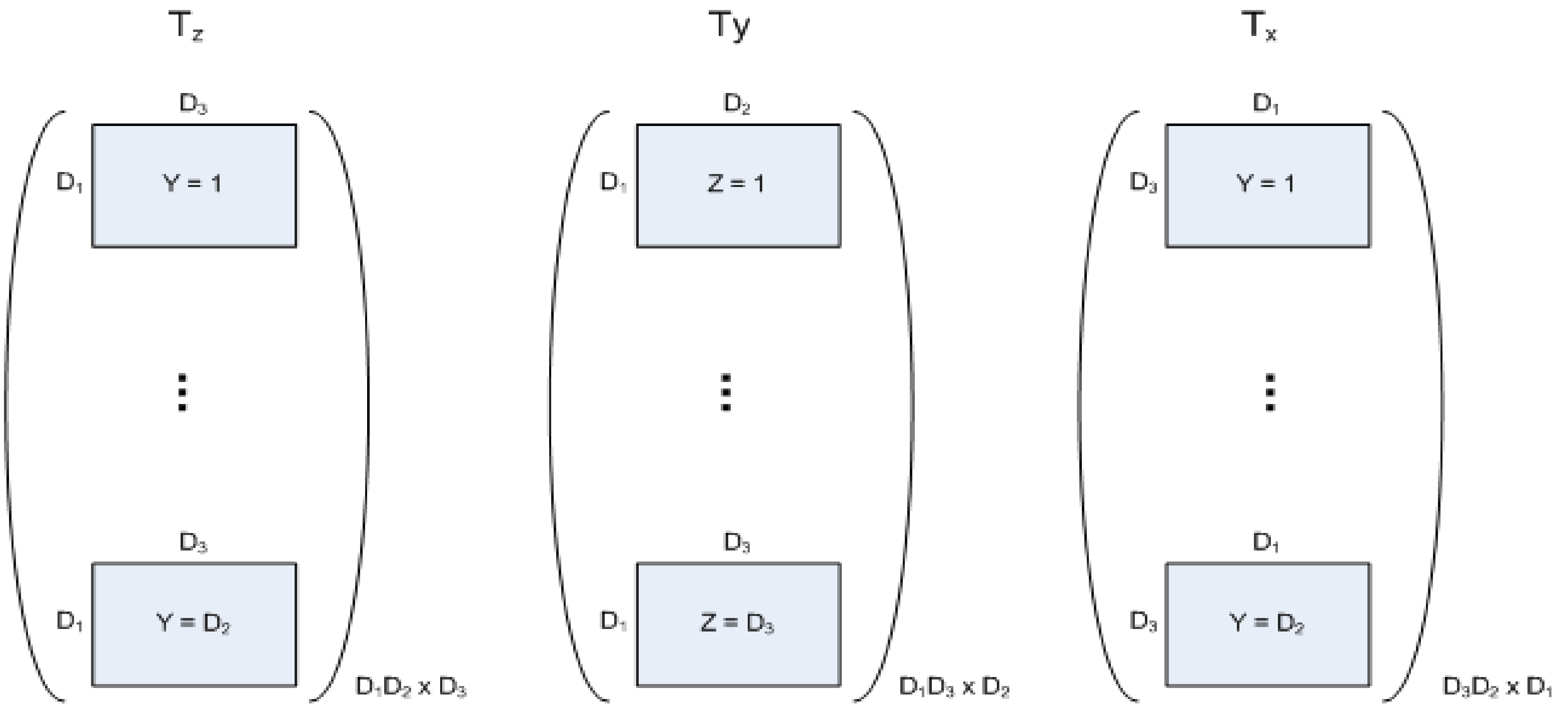
$$\min_{Y} \phi(Y) = \left\| T_y - (X \bullet Z) Y \right\|_F^2$$

- Given Z **and Y, we solve for X by**

$$\min_{X} \phi(X) = \left\| T_x - (Z \bullet Y) X \right\|_F^2$$

Non Negative Tensor Factorization

- Each data matrix, **Tx , Ty , and Tz are permuted and folded** form of the original tensor T , illustrated below.



Non Negative Tensor Factorization

- Given $A \in R^{m \times n} \geq 0$ **and** $W \in R^{m \times k} \geq 0$ **, a semi-NMF problem** is defined as

$$\boxed{\min_{H} \Phi(H) = \|A - WH\|_F^2 \quad \textit{,subject to } \boldsymbol{H \geq 0}}$$

- A modified version of the Projected Gradient Descent (PGD) method is used to solve the Semi-NMF problem. It is basically adding a projection function on top of the regular gradient descent method.

  where the gradient is

$$\boxed{H^{(p+1)} = P_{+}\left[H^{(p)} - \alpha_p \nabla \Phi\left(H^{(p)}\right)\right]}$$

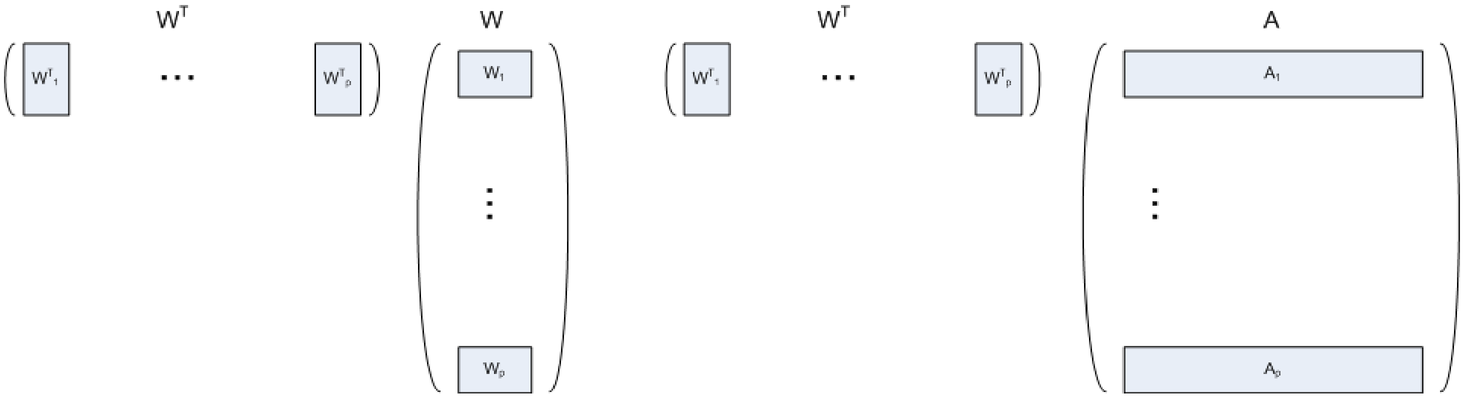
  and P_+ is the projection function

Non Negative Tensor Factorization

- Only need to use two quadratic forms of **W** and **A**, i.e. **W^{T}W** and **W^{T}A**
- Comparing the sizes of two quadratic forms, i.e. $m \times k$ and $m \times n$ with the sizes of **W and A, i.e.** $k \times k$ **and** $k \times n$ **, and** knowing $m, n >> k$, we can save memory required to store these matrices

- A block operation for computing W^{T}W and W^{T}A, where

$$W^T W = \sum_{i=1}^{p} W_i^T W_i \qquad W^T A = \sum_{i=1}^{p} W_i^T A_i$$



Non Negative Tensor Factorization

- Thus, we can partition **X, Y** or **Z** in column blocks and make calls to the PGD subroutine in parallel
- When calling the PGD subroutine, only the quadratic forms $\mathbf{W}^T\mathbf{W}$ and $\mathbf{W}^T\mathbf{A}$ will be used, instead of W and A
- The quadratic forms can also be computed locally by partitioning **W** and **A**, and summed later
- Focus of this PILOT study:

Parallelize the computation of $\mathbf{W}^T\mathbf{A}$

Data Involved

- 6 climate based indices used

Name	Description	Adjustment
sst	sea surface temperature	+273.15
ndvi	normalized difference vegetation index	+0.2
tem	land surface temperature	+273.15
pre	precipitation	
hg500	geopotential height (elevation) for barometric pressure of 500 millibars	+300
hg1000	geopotential height (elevation) for barometric pressure of 1000 millibars	+300

Data Involved

- Preprocessing of data
  - Shifts to enforce non negativity
  - Interpolation to counter sparsity of data
- Each parameter defined by 3-way array
  - Dimension: 720 x 360 x 252
  - 720 - latitude
  - 360 - longitude
  - 252 - month of reading
  - Time dimension: January 1982 – December 2002 (252 months)

Code to be Parallelized

```
function WtA = computeWtA(X,Y,Z,A)
[p k] = size(X);
[q k] = size(Y);
[r k] = size(Z);
WtA = zeros(k,size(A,4));

f{1} = X;
f{2} = Y;
f{3} = Z;

% sort 'p', 'q' and 'r' in ascending  order
[dim c] = sort([p q r]);
f = f(c);
A = reshape(permute(A,[c 4]),[p*q*r size(A,4)]);
M = circDotProd(f{1}, f{2});
```

Code to be Parallelized

```
for i = 1 : dim(3)
    temp = M .* repmat(f{3}(i,:),[size(M,1)  1]);
    WtA = WtA + temp' * A((i-1)*dim(1)
                     *dim(2)+1:i*dim(1)*dim(2),:);
end;
```

Approaches Used

- Parfor Loops
- Distributed Jobs with slicing A
- Load and Save with distributed jobs

Setup

- Cluster of 8 dual core processors (16 workers):
  - *4x Dual Core AMD Opteron(tm) Processor 870 (8-core total, 64-bit) Clock speed: 2 GHz*
- Each approach was tested with subsets of data and finally with the entire data
- Subsets were created based on the time variable. The subsets used were 12, 24 and 180 months
- Execution time was measured using tic/toc function in Matlab

Parfor(Parallel-for) loops

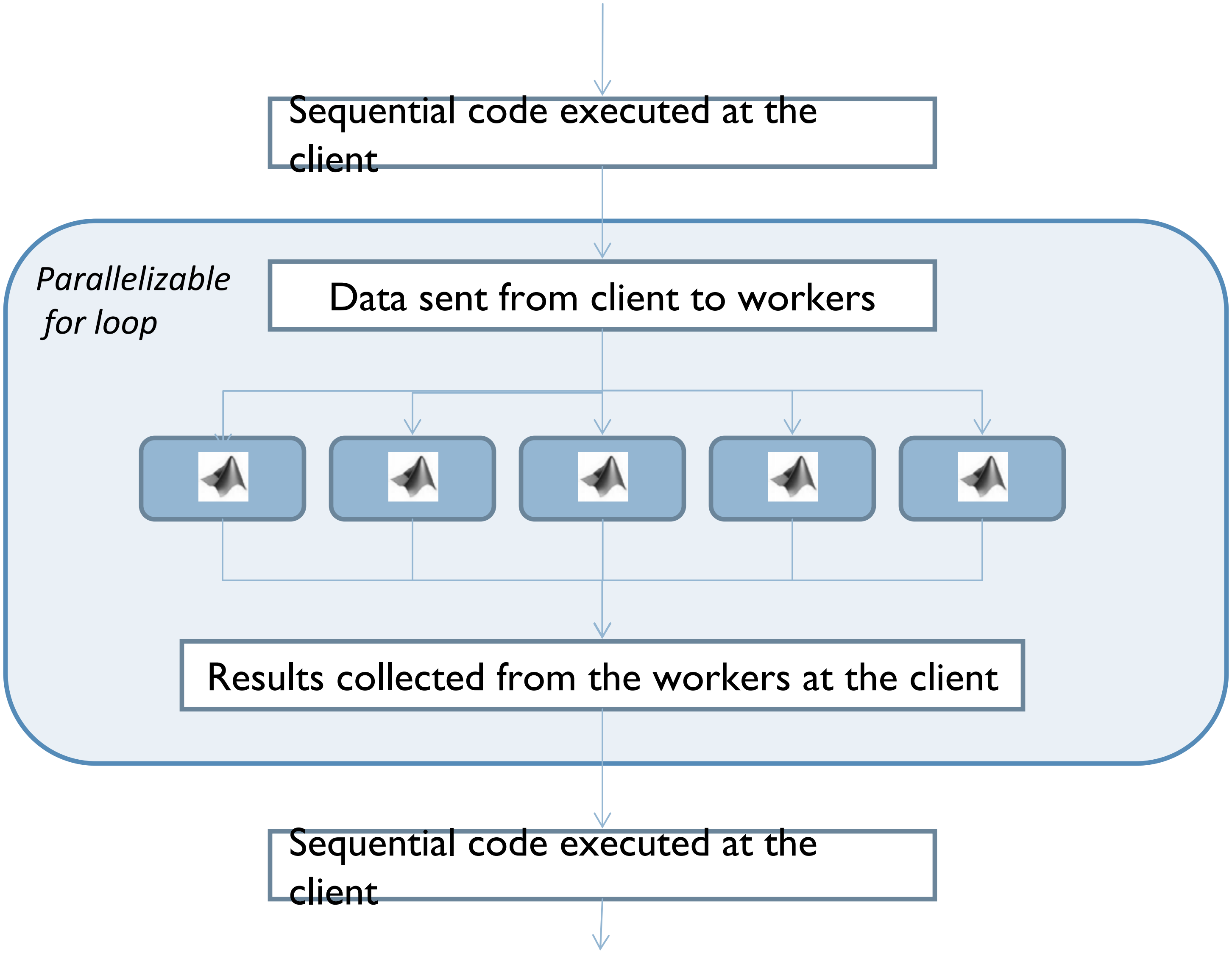
- Part of the loop is executed on client, rest on the worker
- Data sent from client to workers, calculations are performed on workers, results are sent back to client where they are pieced together
- Used when
  - There are loop iterations that take a long time to execute
- Cannot be used when
  - An iteration in the loop depends on other iterations
  - No advantage when there are only simple calculations to be performed in the loop.
- Example

```
x = 0;
parfor i = 1:10
        x = x + i;
end
x
```

Code changes

```
parfor i = 1 : dim(3)
 temp = M .* repmat(f{3}(i,:),[size(M,1)  1]);
WtA = WtA + temp' * A((i-1)*dim(1)
                              *dim(2)+1:i*dim(1)*dim(2),:);
end
```

Code execution



Sequential code executed at the client

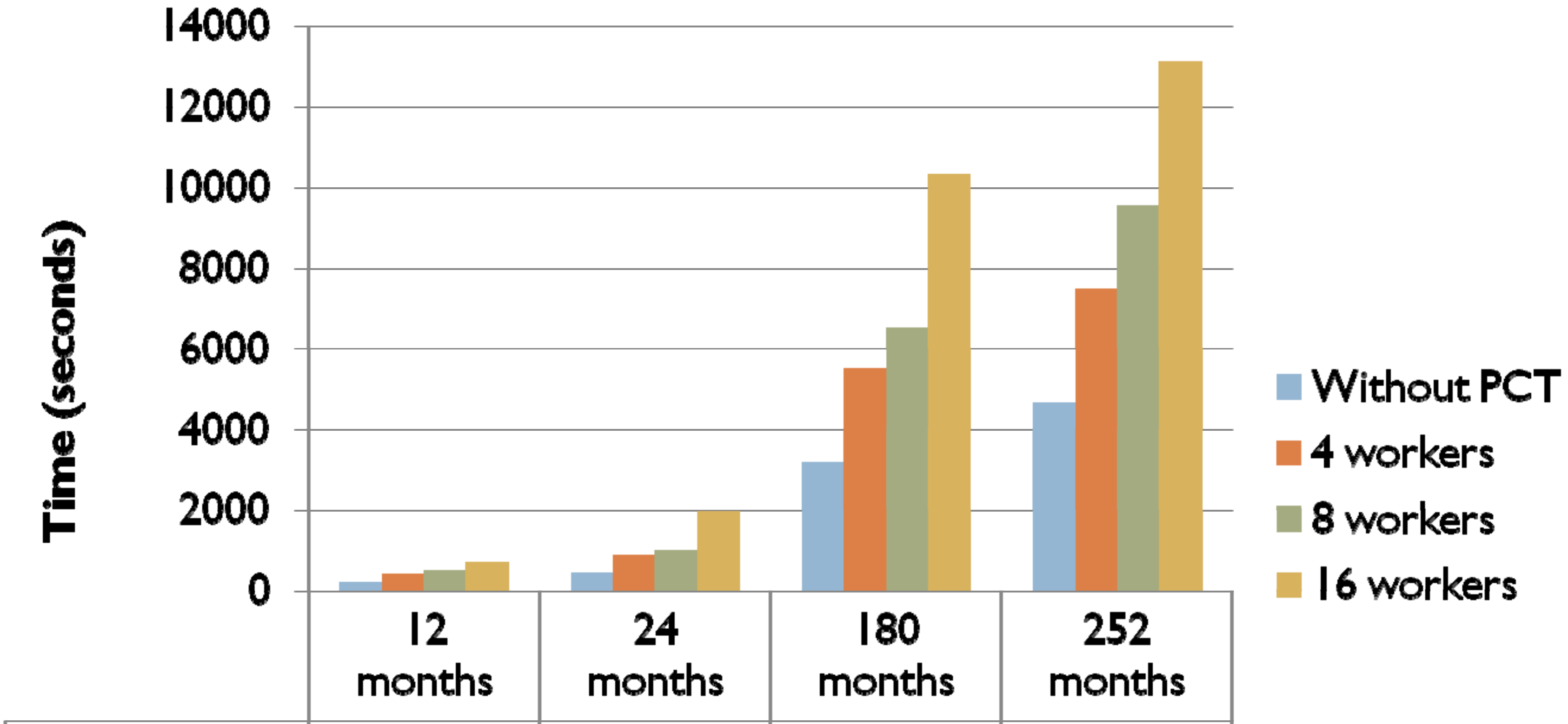
Parallelizable for loop

Data sent from client to workers

Results collected from the workers at the client

Sequential code executed at the client

Execution Times



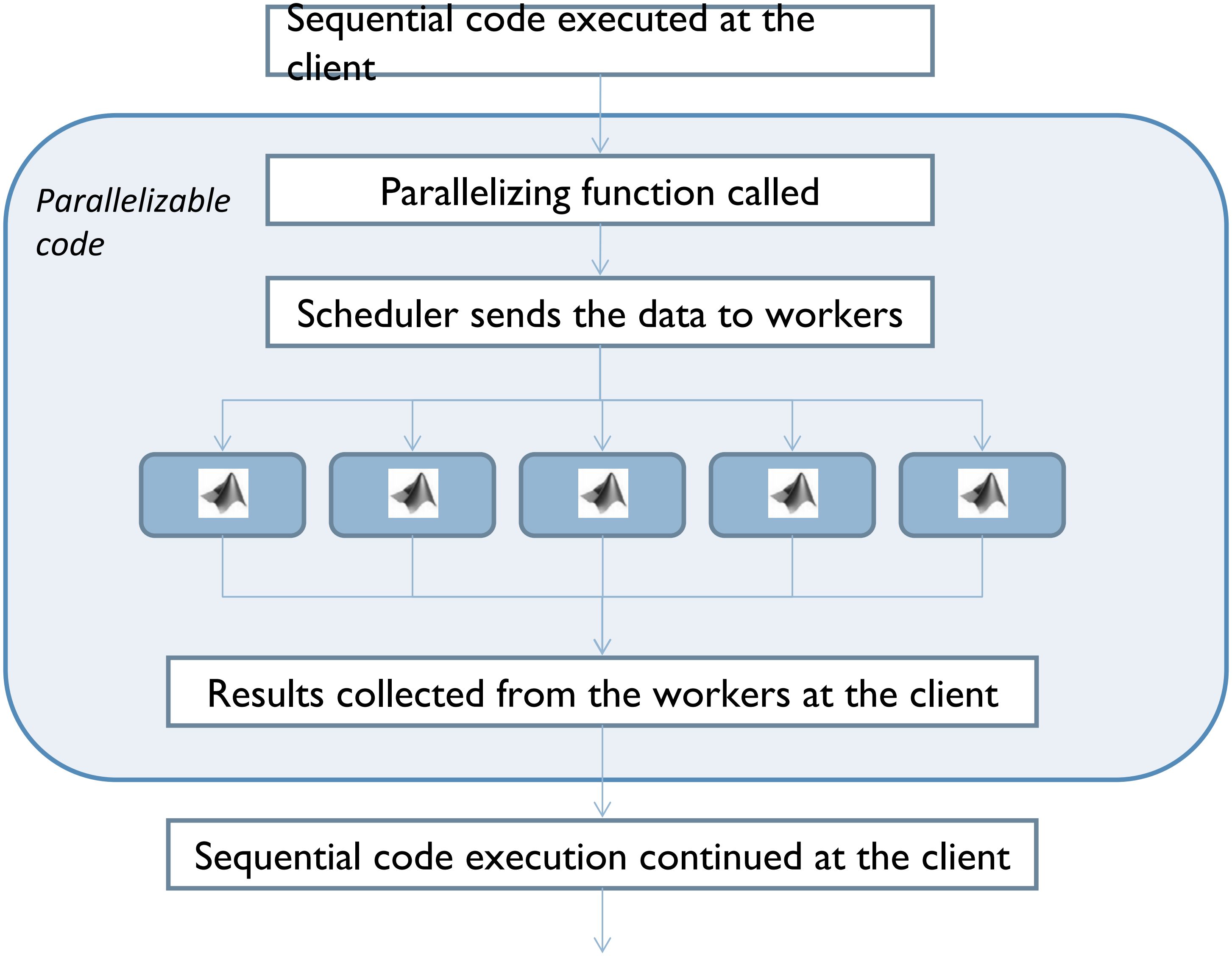
	12 months	24 months	180 months	252 months
🟦 Without PCT	164.372	414.8772	3.16E+03	4642.5
🟧 4 workers	372.2438	853.8555	5500.8	7477.5
🟩 8 workers	468.7778	988.7908	6501.6	9542
🟨 16 workers	674.7118	1.96E+03	10312	13122

Programming Distributed Jobs

In a distributed job:

- Tasks do not directly communicate with each other
- A worker may run several of these tasks in succession
- All tasks perform the same function in a parallel configuration

Code execution



Sequential code executed at the client

Parallelizable code

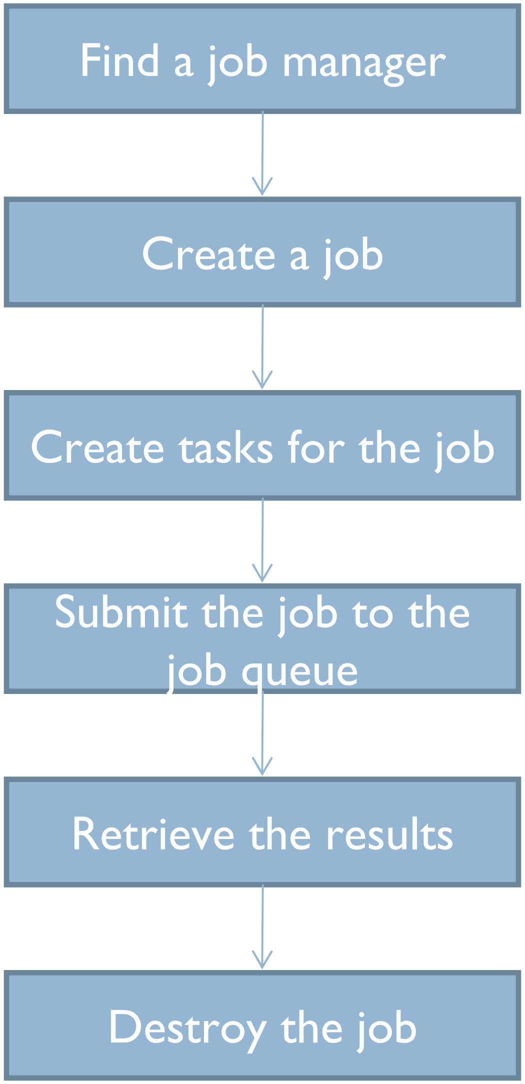
Parallelizing function called

Scheduler sends the data to workers

Results collected from the workers at the client

Sequential code execution continued at the client

Steps in running a distributed job



Find a job manager

Create a job

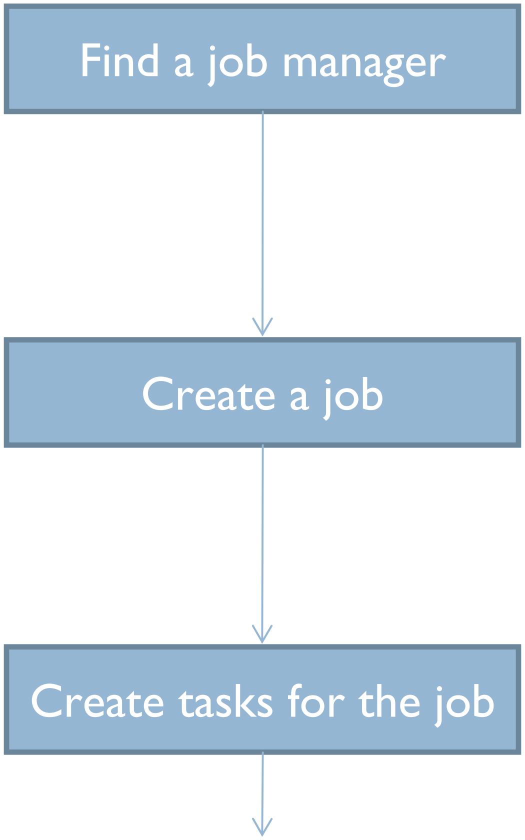
Create tasks for the job

Submit the job to the job queue

Retrieve the results

Destroy the job

Steps in running a distributed job



➢ *findResource* function identifies available job managers and creates an object representing a job manager in your local MATLAB session

Syntax: **jm= findResource('scheduler',‘ type‘,
'jobmanager', 'Name', ‘SamManager',
'lookupURL','localhost');**

➢ Create a job using the available job manager object

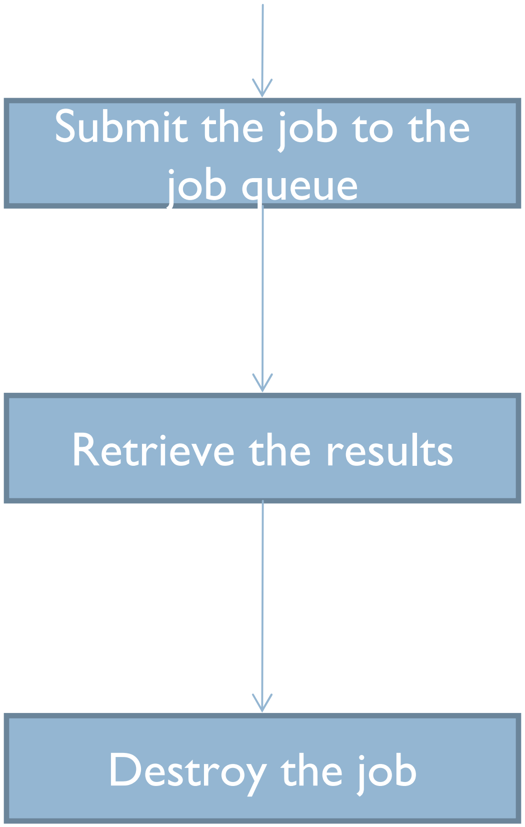
Syntax: **job1 = createJob(jm)**

➢Tasks define the functions to be evaluated by the workers during the running of the job

➢Often, the tasks of a job are all identical

Syntax: **createTask(jobname, functionname, # of outputs, {inputs});**
Eg. **createTask(job1, @rand, 1, {3,3});**

Steps in running a distributed job



Submit the job to the job queue

Retrieve the results

Destroy the job

- To run your job and have its tasks evaluated, you submit the job to the job queue with the submit function

Syntax: **submit(jobname);**

- The results of each task's evaluation are stored in that task object's OutputArguments property as a cell array

Syntax: **results = getAllOutputArguments(jobname);**

- Destroy removes the job object reference object from the local session, and removes the object from the job manager memory

Syntax: **destroy(job)**

Code changes

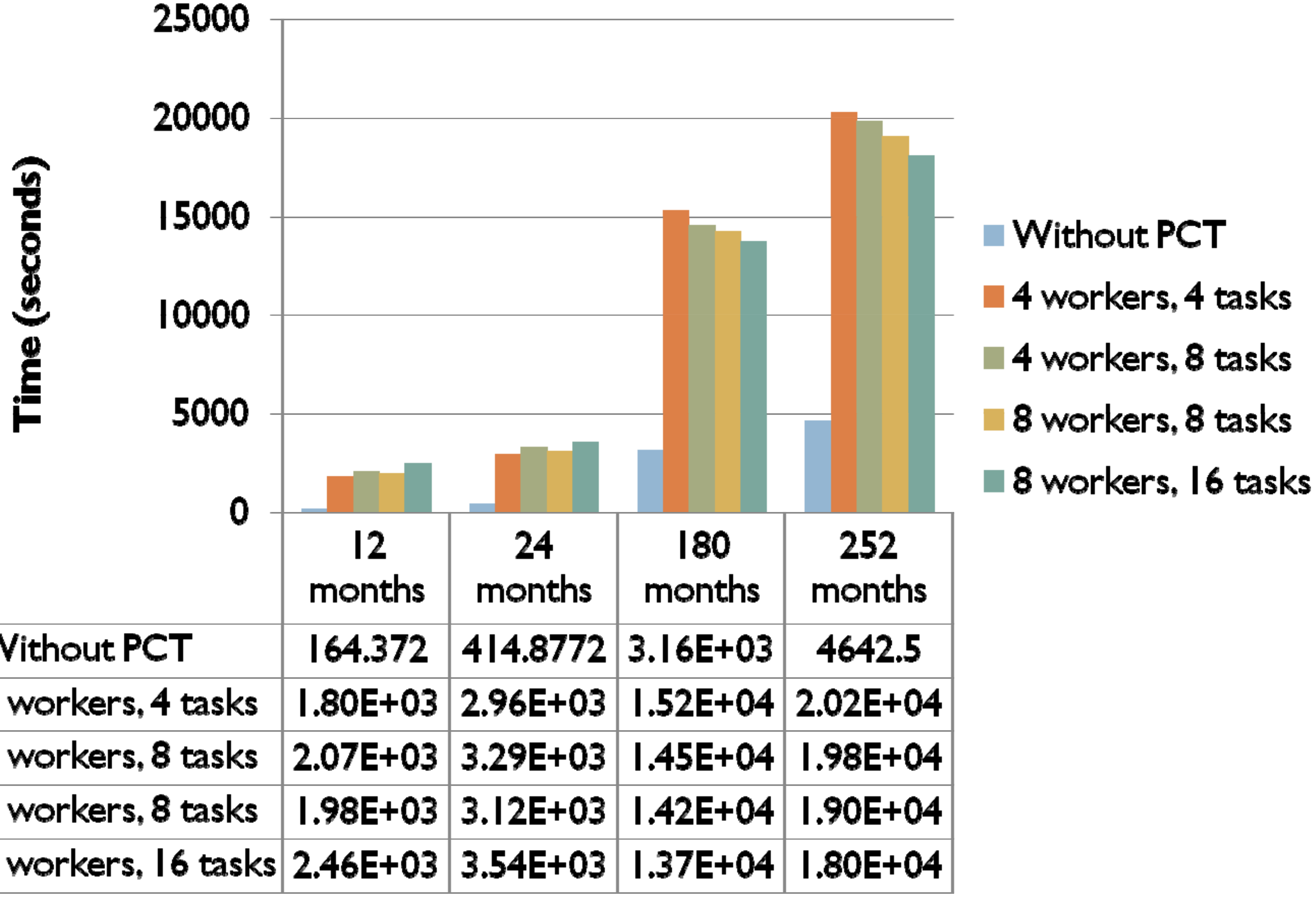
```
function WtA = computeWtA(X,Y,Z,A)
.
.
.
numberOfTasks = 8;
count = dim(3)/numberOfTasks;
jm = findResource('scheduler','type',
    'jobmanager','Name','SamManager',
    'lookupURL','localhost');
job1 = createJob(jm);
set(job1, 'FileDependencies',
    {'WtAparallel.m'});
job1.MinimumNumberOfWorkers = 1;
    job1.MaximumNumberOfWorkers = 4;
```

```
for i = 1 : numberOfTasks
B = A((i-1)*count*dim(1)*dim(2)
            +1:(i*count)*dim(1)*dim(2),:);
t(i) = createTask(job1, @WtAparallel, 1,
    {M,f{3},i, count, dim(1),dim(2),B, WtA});
end
submit(job1);
waitForState(job1);
results = getAllOutputArguments(job1);
for j = 1: numberOfTasks
      WtA = WtA + cell2mat(results(j));
end;
 destroy(job1);
```

Code changes

```
function finalWtA= WtAparallel(M,f,i,count,d1,d2,B, WtA)
finalWtA= WtA;
for k = 1 : count,
    temp = M .* repmat(f(k,:),[size(M,1) 1]);
    finalWtA= finalWtA+ (temp' * B((k-1) * d1 * d2 + 1:k*d1 * d2, :));
end;
```

Execution Times with Distributed Jobs

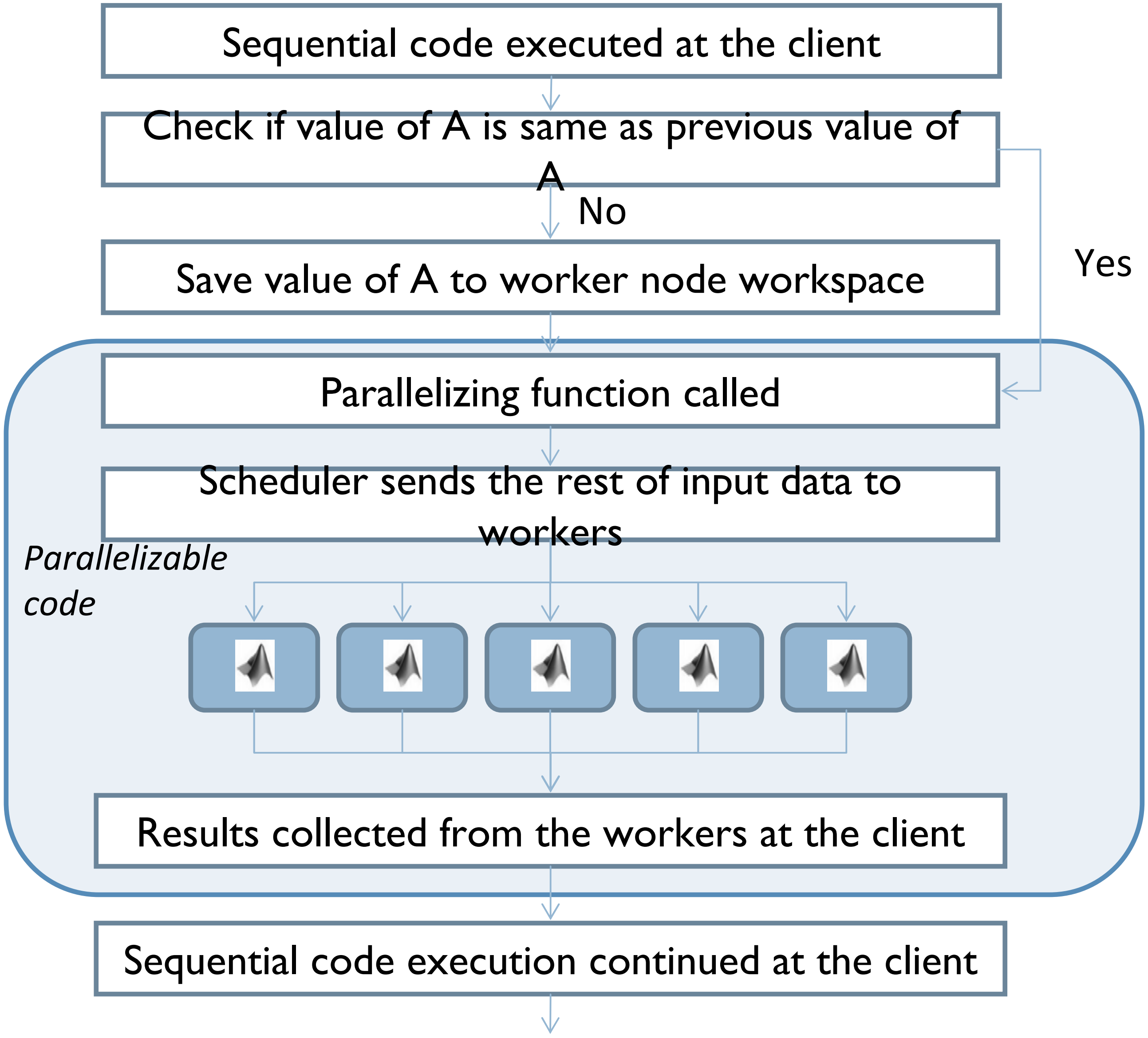


	12 months	24 months	180 months	252 months
🟦 Without PCT	164.372	414.8772	3.16E+03	4642.5
🟧 4 workers, 4 tasks	1.80E+03	2.96E+03	1.52E+04	2.02E+04
🟩 4 workers, 8 tasks	2.07E+03	3.29E+03	1.45E+04	1.98E+04
🟨 8 workers, 8 tasks	1.98E+03	3.12E+03	1.42E+04	1.90E+04
🟩 8 workers, 16 tasks	2.46E+03	3.54E+03	1.37E+04	1.80E+04

Load and Save with Distributed Jobs

- Size of matrix A is very large and linear. For entire dataset the size of A is 65318400 x1
- In Distributed Jobs, A was being passed to the worker node every time a task was created
- This created huge overheads
- In this approach, A is saved to the local workspace of the node, prior to task creation and is reloaded only when there is a change in the value of A
- This minimizes the data overhead every time a task is created

Load and Save code execution at client



Sequential code executed at the client

Check if value of A is same as previous value of A

No

Yes

Save value of A to worker node workspace

Parallelizing function called

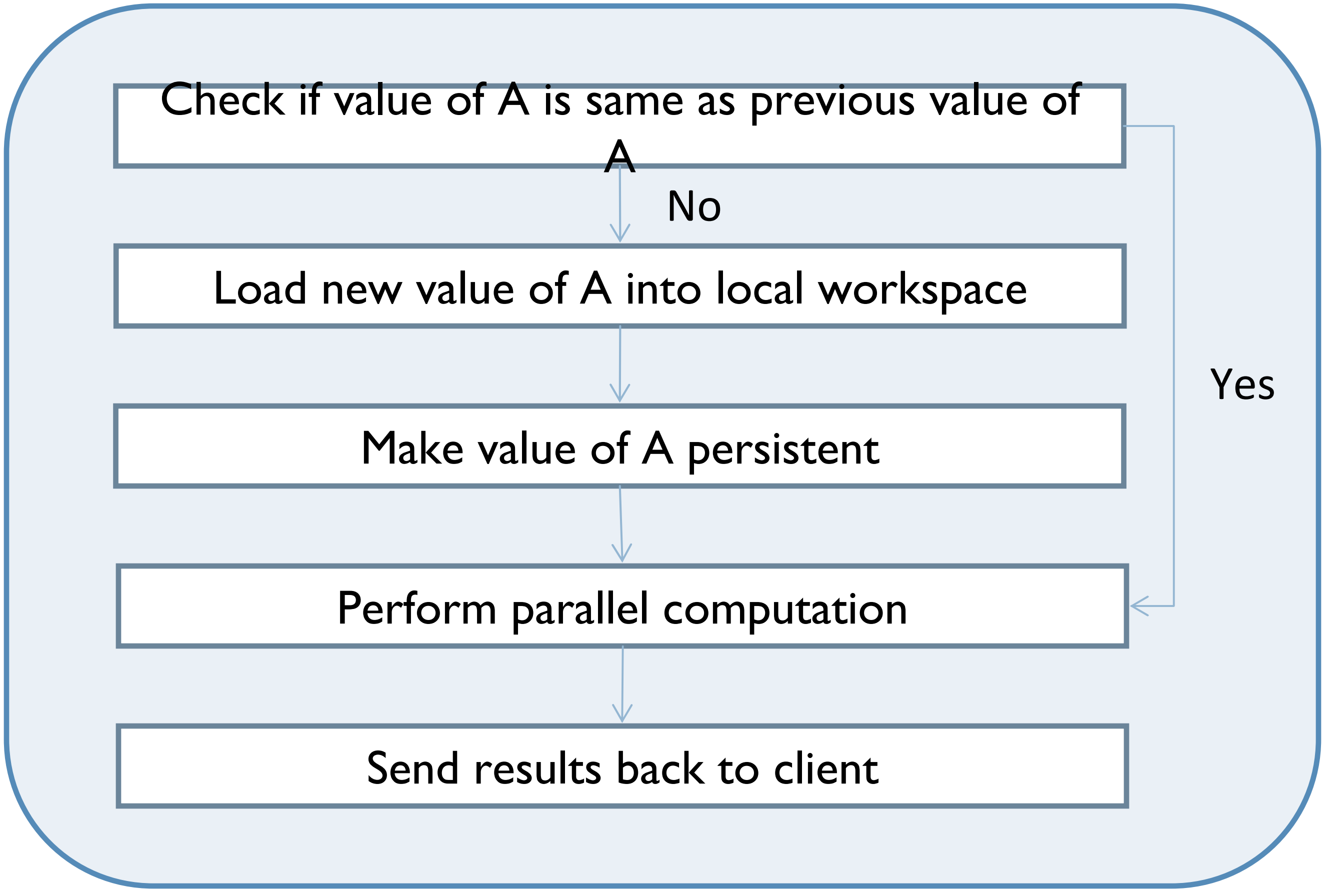
Scheduler sends the rest of input data to workers

Parallelizable code

Results collected from the workers at the client

Sequential code execution continued at the client

Load and save code execution at worker node



Check if value of A is same as previous value of A
No
Load new value of A into local workspace
Make value of A persistent
Yes
Perform parallel computation
Send results back to client

Code changes

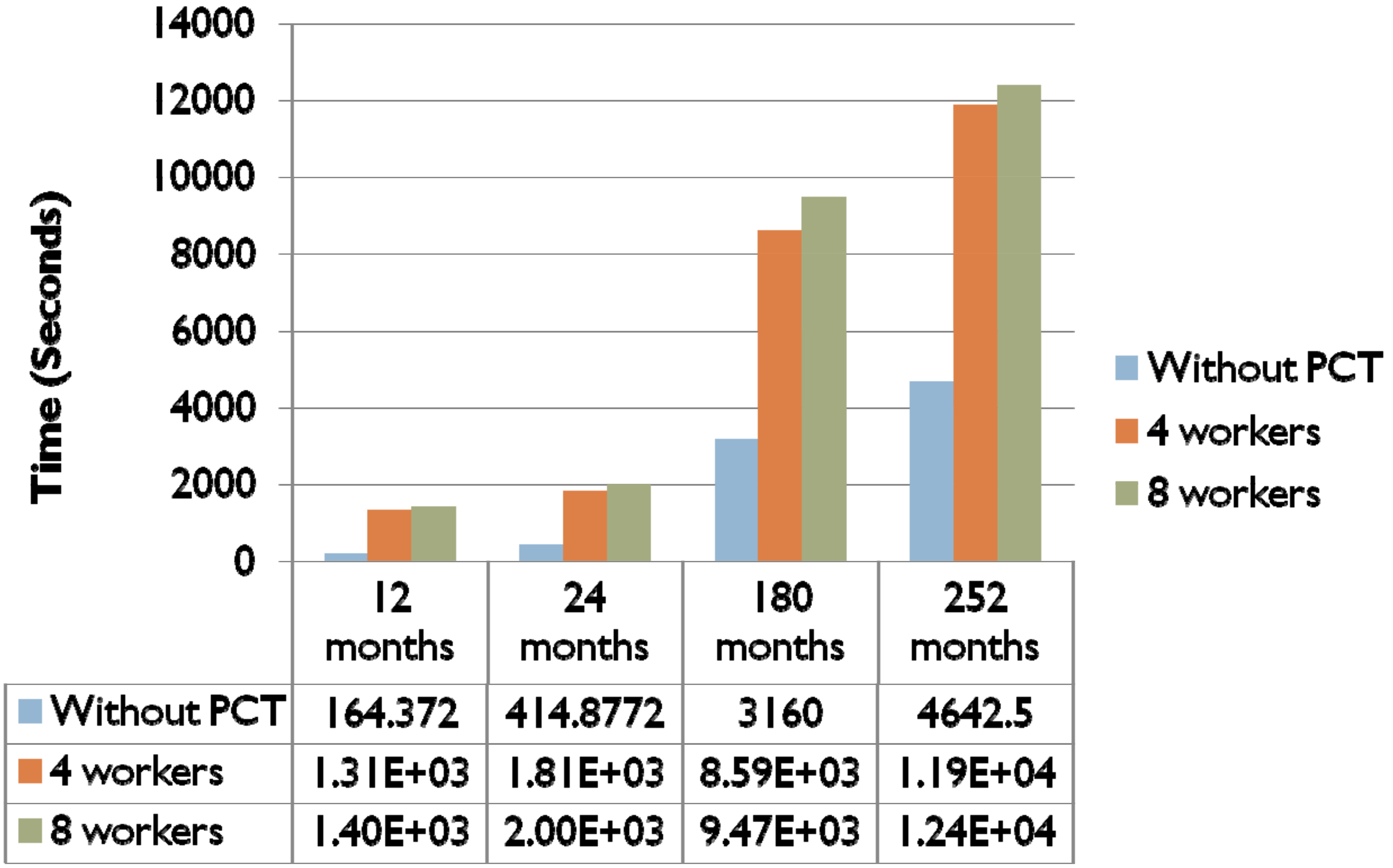
```
function WtA = computeWtA(X,Y,Z,A)
.
.
.
A = reshape(permute(A,[c 4]),[p*q*r
    size(A,4)]);
oldA = load('array_a.mat','A');
if isequal(A,oldA)
   flagA = 0;
else
   flagA = 1;
   save('array_a.mat','A');
end;
M = circDotProd(f{1}, f{2});
numberOfTasks = 8;
count = dim(3)/numberOfTasks;
jm =
    findResource('scheduler','type','jobmanage
    r','Name','SamManager','lookupURL','local
    host');
job1 = createJob(jm);
```

```
set(job1,'FileDependencies',{'WtAparallel.m'
     'array_a.mat'});

job1.MinimumNumberOfWorkers = 1;
job1.MaximumNumberOfWorkers = 4;
for i = 1 : numberOfTasks
     createTask(job1, @WtAparallel, 1, {M,f{3},i,
     count, dim(1),dim(2), flagA, WtA});
 end
submit(job1);
 waitForState(job1);
results = getAllOutputArguments(job1);
for j = 1:numberOfTasks
       WtA = WtA + cell2mat(results(j));
end;
destroy(job1);
```

Code Changes

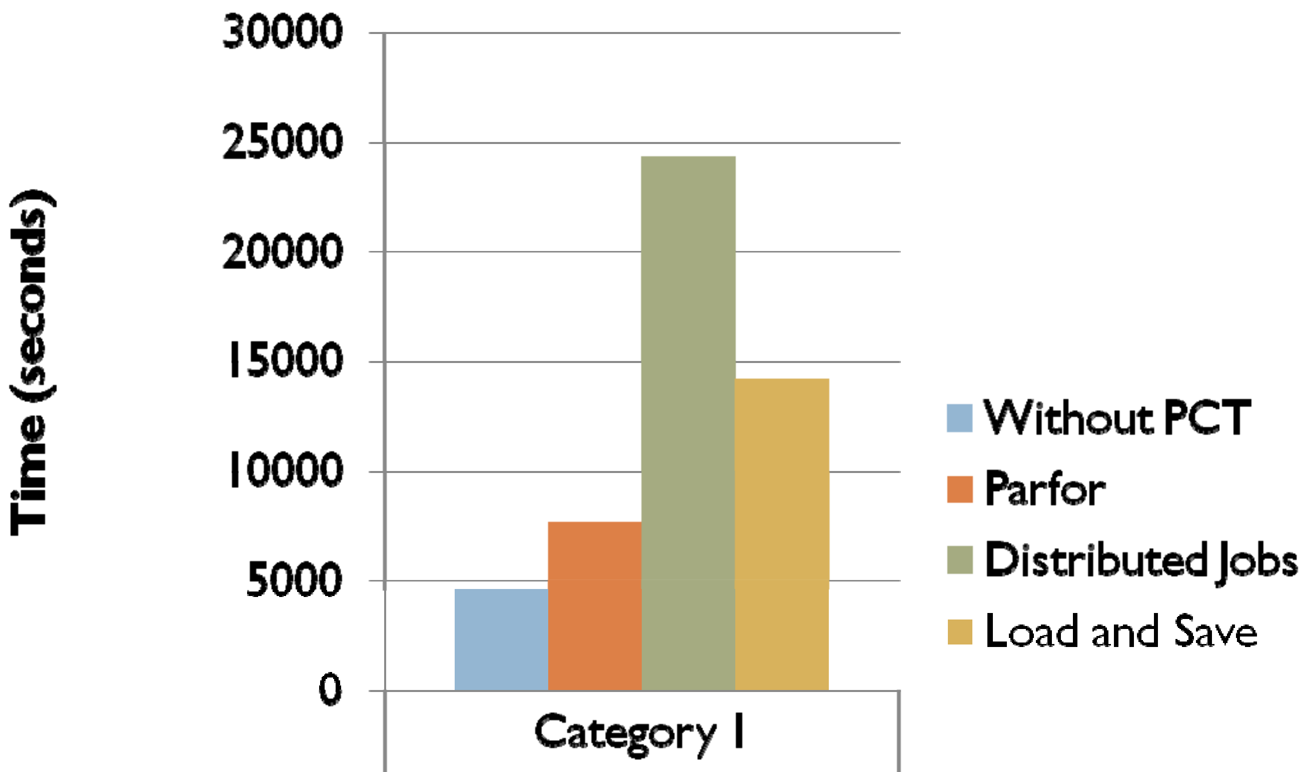
```
function finalWtA= WtAparallel(M,f,i,count,d1,d2, flagA, WtA)
if flagA==1
    persistent A;
    load('array_a.mat', 'A');
end;

finalWtA = WtA;
for k = 1 : count,
    l = i+k;
    temp = M .* repmat(f(l,:),[size(M,1) 1]);
    finalWtA = finalWtA+ (temp' * A((l-1) * d1 * d2 + 1:l*d1 * d2, :));
end;
```

Execution Times with Load and Save



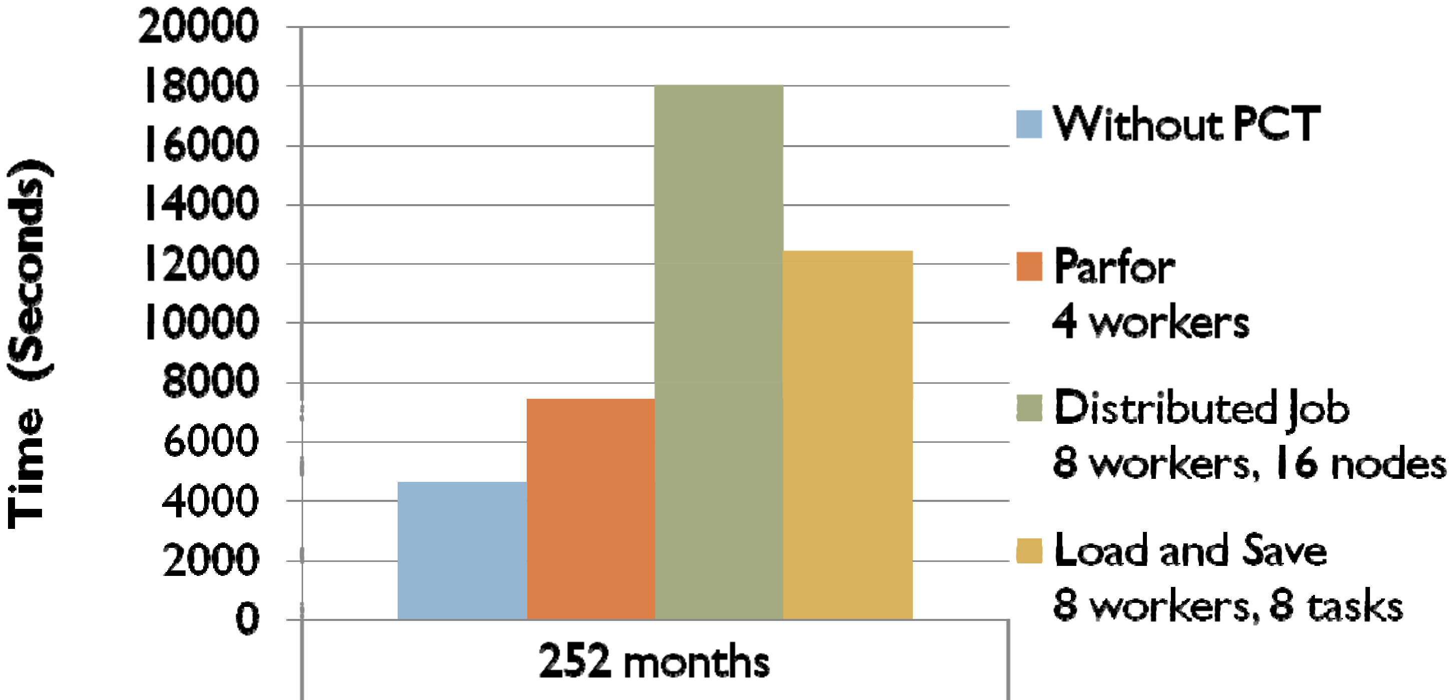
	12 months	24 months	180 months	252 months
🟦 Without PCT	164.372	414.8772	3160	4642.5
🟧 4 workers	1.31E+03	1.81E+03	8.59E+03	1.19E+04
🟩 8 workers	1.40E+03	2.00E+03	9.47E+03	1.24E+04

Overheads



	Category 1
🟦 Without PCT	4642.5
🟧 Parfor	7674.4
🟩 Distributed Jobs	24356
🟨 Load and Save	14194.4

Overall Comparison



	252 months
🟦 Without PCT	4642.5
🟧 Parfor 4 workers	7447.5
🟩 Distributed Job 8 workers, 16 nodes	18040
🟨 Load and Save 8 workers, 8 tasks	12363

Conclusions drawn

- Parfor loops
  - By far, the best performance among the three methods used
  - The easiest to use in terms of code modification
  - Data overhead is minimal when compared to other two methods

Conclusions drawn

- Distributed jobs
  - Except for the load and save method, there is no way of controlling the workspace of worker node
  - Workers cannot share a workspace with the client, hence all input must be available to all workers
  - Cannot determine node – task allocation, it is done by the scheduler
  - Inputs have to be bound to the task at the time of creation, cannot be bound to the task at a later point of time
  - Task execution is not staggered i.e. there is no time lag between the start of tasks at worker nodes

Conclusions drawn

- Load and Save
  - Can bind a variable to a node's workspace for the length of the job, this eliminates the need to send it as a part of input while creating the task
  - The "persistent" function saves the value of a variable for the duration of the job

Conclusions drawn

- Parallel Computing Toolbox – Overall
  - Parallel Computing Toolbox does not lend itself to linear inputs and relatively less complex parallel code
  - On experimental runs with more regular square matrix data there was significant improvement over sequential execution of code
    - Eg. FFT and InverseFFT code run on two matrices of size 500*500 and 900 * 900
    - Distributed Jobs with 8 worker nodes: 179.5767s
    - Sequential execution of code: 456.4300s

References

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