A Finite State Machine Approach to Cluster Identification Using the Hoshen-Kopelman Algorithm

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Objective

Cluster Identification

- Want to find and identify homogeneous patches in a 2D matrix, where:
  - Cluster membership defined by adjacency
    - No need for distance function
  - Sequential cluster IDs not necessary
- Common task in analysis of geospatial data (landscape maps)

Hoshen-Kopelman Algorithm

Overview

- Assigns unique IDs to homogeneous regions in a lattice
- Handles only one *target class* at a time
  - Lattice preprocessing needed to filter out unwanted classes
- Single-pass cluster identification
  - Second pass to relabel temporary IDs, but not strictly necessary
- 2-D lattice represented as matrix herein

Hoshen-Kopelman Algorithm

Data structures

- Matrix
  - Preprocessed to replace target class with *-1*, everything else with *0*
- Cluster ID/size array (“csize”)
  - Indexing begins at 1
  - Index represents cluster ID
  - Positive values indicate cluster size
    - *Proper* cluster label
  - Negative values provide ID redirection
    - *Temporary* cluster label

Hoshen-Kopelman Algorithm

csize array

- + values: cluster size
  - Cluster 2 has 8 members
- - values: ID redirection
  - Cluster 4 is the same as cluster 1, same as cluster 3
    - Cluster 4/1/3 has 5 members
  - Redirection allowed for noncircular, recursive path for finite number of steps

1	-3
2	8
3	5
4	-1
5	4
6	1
7	0

Hoshen-Kopelman Algorithm

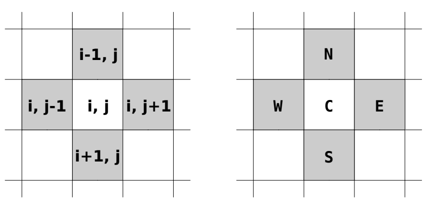
Clustering procedure

- Matrix traversed row-wise
- If current cell nonzero
  - Search for nonzero (target class) neighbors
  - If no nonzero neighbors found ...
    - Give cell new label
  - Else ...
    - Find proper labels *K* of nonzero neighbor cells
    - min(*K*) is the new proper label for current cell and nonzero neighbors

Hoshen-Kopelman Algorithm

Nearest-Four Neighborhood

- North/East/West/South neighbors
- Used in classic HK implementations
- Of the four neighbors, only N/W have been previously labeled at any given time



Hoshen-Kopelman Algorithm

Nearest-4 HK in action...

-1	0	-1	0	0	-1	-1	0
-1	0	-1	0	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1
-1	-1	0	-1	0	-1	0	-1
-1	0	0	0	-1	0	-1	0
-1	0	0	-1	-1	0	0	0
0	0	-1	-1	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1

1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0

- Matrix has been preprocessed
  - Target class value(s) replaced with *-1*, all others with *0*

Hoshen-Kopelman Algorithm

Nearest-4 HK in action...

1	0	2	0	0	3	3	0
-1	0	-1	0	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1
-1	-1	0	-1	0	-1	0	-1
-1	0	0	0	-1	0	-1	0
-1	0	0	-1	-1	0	0	0
0	0	-1	-1	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1

1	1
2	1
3	2
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0

- First row, two options:
  - Add top buffer row of zeros, OR
  - Ignore N neighbor check

Hoshen-Kopelman Algorithm

Nearest-4 HK in action...

1	0	2	0	0	3	3	0
-1	0	-1	0	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1
-1	-1	0	-1	0	-1	0	-1
-1	0	0	0	-1	0	-1	0
-1	0	0	-1	-1	0	0	0
0	0	-1	-1	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1

1	1
2	1
3	2
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0

Hoshen-Kopelman Algorithm

Nearest-4 HK in action...

1	0	2	0	0	3	3	0
1	0	-1	0	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1
-1	-1	0	-1	0	-1	0	-1
-1	0	0	0	-1	0	-1	0
-1	0	0	-1	-1	0	0	0
0	0	-1	-1	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1

1	2
2	1
3	2
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0

Hoshen-Kopelman Algorithm

Nearest-4 HK in action...

1	0	2	0	0	3	3	0
1	0	-1	0	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1
-1	-1	0	-1	0	-1	0	-1
-1	0	0	0	-1	0	-1	0
-1	0	0	-1	-1	0	0	0
0	0	-1	-1	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1

1	2
2	1
3	2
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0

Hoshen-Kopelman Algorithm

Nearest-4 HK in action...

1	0	2	0	0	3	3	0
1	0	2	0	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1
-1	-1	0	-1	0	-1	0	-1
-1	0	0	0	-1	0	-1	0
-1	0	0	-1	-1	0	0	0
0	0	-1	-1	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1

1	2
2	2
3	2
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0

Hoshen-Kopelman Algorithm

Nearest-4 HK in action...

1	0	2	0	0	3	3	0
1	0	2	0	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1
-1	-1	0	-1	0	-1	0	-1
-1	0	0	0	-1	0	-1	0
-1	0	0	-1	-1	0	0	0
0	0	-1	-1	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1

1	2
2	2
3	2
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0

Hoshen-Kopelman Algorithm

Nearest-4 HK in action...

1	0	2	0	0	3	3	0
1	0	2	0	4	-1	0	-1
0	0	-1	-1	-1	-1	0	-1
-1	-1	0	-1	0	-1	0	-1
-1	0	0	0	-1	0	-1	0
-1	0	0	-1	-1	0	0	0
0	0	-1	-1	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1

1	2
2	2
3	2
4	1
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0

Hoshen-Kopelman Algorithm

Nearest-4 HK in action...

1	0	2	0	0	3	3	0
1	0	2	0	4	3	0	-1
0	0	-1	-1	-1	-1	0	-1
-1	-1	0	-1	0	-1	0	-1
-1	0	0	0	-1	0	-1	0
-1	0	0	-1	-1	0	0	0
0	0	-1	-1	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1

1	2
2	2
3	4
4	-3
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0

Hoshen-Kopelman Algorithm

Nearest-4 HK in action...

1	0	2	0	0	3	3	0
1	0	2	0	4	3	0	-1
0	0	-1	-1	-1	-1	0	-1
-1	-1	0	-1	0	-1	0	-1
-1	0	0	0	-1	0	-1	0
-1	0	0	-1	-1	0	0	0
0	0	-1	-1	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1

1	2
2	2
3	4
4	-3
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0

Hoshen-Kopelman Algorithm

Nearest-4 HK in action...

1	0	2	0	0	3	3	0
1	0	2	0	4	3	0	5
0	0	-1	-1	-1	-1	0	-1
-1	-1	0	-1	0	-1	0	-1
-1	0	0	0	-1	0	-1	0
-1	0	0	-1	-1	0	0	0
0	0	-1	-1	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1

1	2
2	2
3	4
4	-3
5	1
6	0
7	0
8	0
9	0
10	0
11	0
12	0

Hoshen-Kopelman Algorithm

Nearest-4 HK in action...

1	0	2	0	0	3	3	0
1	0	2	0	4	3	0	5
0	0	2	2	2	2	0	5
-1	-1	0	-1	0	-1	0	-1
-1	0	0	0	-1	0	-1	0
-1	0	0	-1	-1	0	0	0
0	0	-1	-1	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1

1	2
2	10
3	-2
4	-3
5	2
6	0
7	0
8	0
9	0
10	0
11	0
12	0

Hoshen-Kopelman Algorithm

Nearest-4 HK in action...

1	0	2	0	0	3	3	0
1	0	2	0	4	3	0	5
0	0	2	2	2	2	0	5
6	6	0	2	0	2	0	5
-1	0	0	0	-1	0	-1	0
-1	0	0	-1	-1	0	0	0
0	0	-1	-1	-1	-1	0	-1
0	0	-1	-1	-1	-1	0	-1

1	2
2	12
3	-2
4	-3
5	3
6	2
7	0
8	0
9	0
10	0
11	0
12	0

Hoshen-Kopelman Algorithm

Nearest-4 HK in action...

1	0	2	0	0	3	3	0
1	0	2	0	4	3	0	5
0	0	2	2	2	2	0	5
6	6	0	2	0	2	0	5
6	0	0	0	7	0	8	0
6	0	0	9	7	0	0	0
0	0	10	7	7	7	0	11
0	0	7	7	7	7	0	11

1	2
2	12
3	-2
4	-3
5	3
6	3
7	11
8	1
9	-7
10	-7
11	2
12	0

- Skipping ahead

Hoshen-Kopelman Algorithm

Nearest-4 HK in action...

1	0	2	0	0	2	2	0
1	0	2	0	2	2	0	5
0	0	2	2	2	2	0	5
6	6	0	2	0	2	0	5
6	0	0	0	7	0	8	0
6	0	0	7	7	0	0	0
0	0	7	7	7	7	0	11
0	0	7	7	7	7	0	11

1	2
2	12
3	-2
4	-3
5	3
6	3
7	11
8	1
9	-7
10	-7
11	2
12	0

- Optional second pass to relabel cells to their proper labels

Hoshen-Kopelman Algorithm

Nearest-Eight Neighborhood

- NW, N, NE, E, SE, S, SW, W
- When examining a cell, compare to W, NW, N, NE neighbors

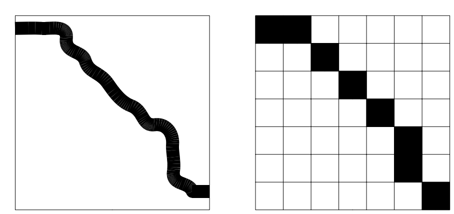
i-1, j-1	i-1, j	i-1, j+1
i, j-1	i, j	i, j+1
i+1, j-1	i+1, j	i+1, j+1

NW	N	NE
W	C	E
SW	S	SE

Hoshen-Kopelman Algorithm

Nearest-Eight Neighborhood

- Sometimes more appropriate in landscape analysis
- Rasterization can segment continuous features if only using nearest-four neighborhood



Hoshen-Kopelman Algorithm

Nearest-4 vs. Nearest-8 Results

1	0	2	0	0	2	2	0
1	0	2	0	2	2	0	5
0	0	2	2	2	2	0	5
6	6	0	2	0	2	0	5
6	0	0	0	7	0	8	0
6	0	0	7	7	0	0	0
0	0	7	7	7	7	0	11
0	0	7	7	7	7	0	11

1	0	2	0	0	2	2	0
1	0	2	0	2	2	0	2
0	0	2	2	2	2	0	2
2	2	0	2	0	2	0	2
2	0	0	0	2	0	2	0
2	0	0	2	2	0	0	0
0	0	2	2	2	2	0	5
0	0	2	2	2	2	0	5

UNION-FIND Algorithm

Disjoint-Set Data Structure

- Maintains collection of non-overlapping sets of objects
- Each set identifiable by a single *representative* object
  - Rep. may change as set changes, but remains the same as long as set unchanged
- Disjoint-set forest is a type of D-S data structure with sets represented by rooted trees
  - Root of tree is representative

UNION-FIND Algorithm

Disjoint-Set Data Structure Operations

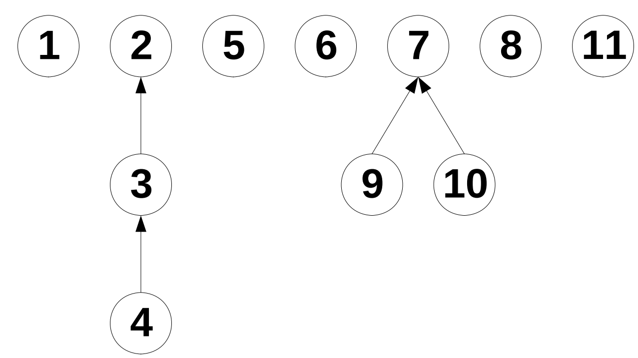
- MAKE-SET(*x*)
  - Creates a new set whose only member is *x*
- UNION(*x*, *y*)
  - Combines the two sets containing objects *x* and *y*
- FIND-SET(*x*)
  - Returns the representative of the set containing object *x*
- An algorithm that performs these ops is known as a UNION-FIND algorithm

UNION-FIND Algorithm

HK relation to UNION-FIND

- csize array may be viewed as a disjoint-set forest

1	2
2	12
3	-2
4	-3
5	3
6	3
7	11
8	1
9	-7
10	-7
11	2
12	0



UNION-FIND Algorithm

HK relation to UNION-FIND

- Implementation of UNION-FIND operations
  - MAKE-SET: When a cell is given a new label and new cluster is formed
  - UNION: When two clusters are merged
  - FIND-SET: Also when two clusters are merged (must determine that the proper labels of the two clusters differ)

UNION-FIND Algorithm

Heuristics to improve UNION-FIND

- Path compression
  - Used in FIND-SET to set each node's parent link to the root/representative node
  - FIND-SET becomes two-pass method
    1) Follow parent path of *x* to find root node
    2) Traverse back down path and set each node's parent pointer to root node

UNION-FIND Algorithm

Heuristics to improve UNION-FIND

- Union by rank
  - Goal: When performing UNION, set root of smaller tree to point to root of larger tree
  - Size of trees not explicitly tracked; rather, a *rank* metric is maintained
  - Rank is upper bound on height of a node
  - MAKE-SET: Set rank of node to 0
  - UNION: Root with higher node becomes parent; in case of tie, choose arbitrarily and increase winner's rank by 1

UNION-FIND Algorithm

Applying these heuristics to HK

- Original HK did not use either heuristic
- Previous FSM implementation (Constantin, et al.) used only path compression
- Implementation in this study uses path compression and union by cluster size
  - U by cluster size: Similar to U by rank, but considers size of cluster represented by tree, not size of tree itself
    - Reduces the number of relabeling ops in 2^{nd} pass

Finite State Machines

Computational model composed of:

- Set of states
  - Each state stores some form of input history
- Input alphabet (set of symbols)
  - Input is read by FSM sequentially
- State transition rules
  - Next state determined by current state and current input symbol
  - Need rule for every state/input combination

Finite State Machines

Formal definition: $(S, \Sigma, \delta, q_0, F)$

- S: Set of states
- Σ: Input alphabet
  - Input is read by FSM sequentially
- δ: State transition rules
  - $(\delta: S \times \Sigma \rightarrow S)$
- q_0: Starting state
- F: Set of final states

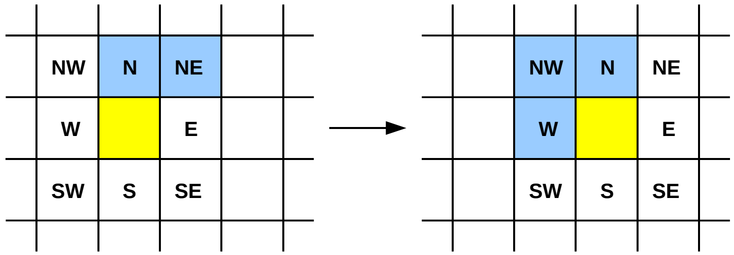
Nearest-8 HK with FSM

Why apply FSM to Nearest-8 HK?

- Want to retain short-term knowledge on still relevant, previously examined cells
  - Helps avoid costly memory accesses
- Recall from Nearest-8 HK that the W, NW, N, NE neighbors' values are checked when examining each cell
  - *(only when the current cell is nonzero!)*

Nearest-8 HK with FSM

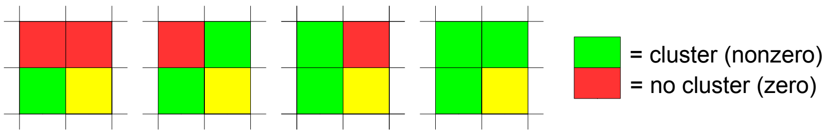
- Note that a cell and its N, NE neighbors are next cell's W, NW, N neighbors
- Encapsulate what is known about current cell and N, NE neighbors into next state
  - Number of neighbor comparisons can be reduced by up to 75%



Nearest-8 HK with FSM

Let's define our state space...

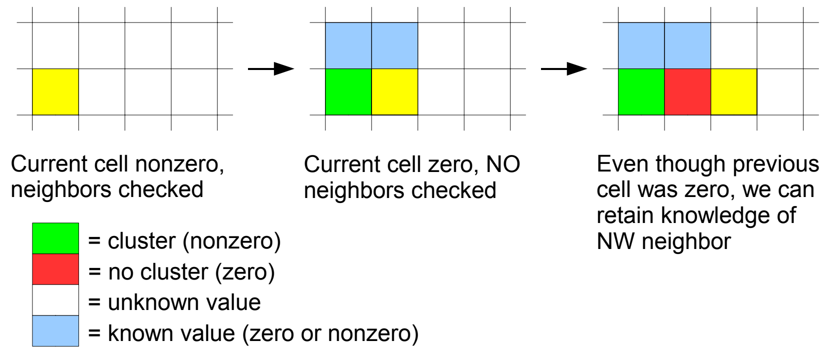
- Current cell value is *always* checked, thus always encapsulated in the next state
- Assume current cell value is nonzero
  - N, NE neighbor values are checked (along with NW, but that's irrelevant for next cell)
  - This produces four possible states when examining the next cell:



Nearest-8 HK with FSM

And if current cell is zero?

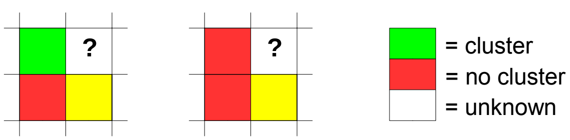
- Neighbor values are not checked
  - But some neighbor knowledge may still be retained. Consider:



Nearest-8 HK with FSM

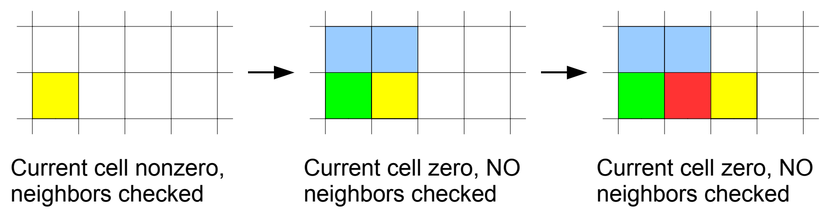
So, after a single zero value...

- We can still retain knowledge of NW neighbor
- This produces two more states:

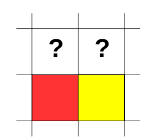


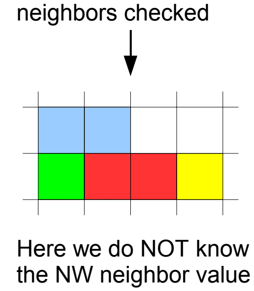
Nearest-8 HK with FSM

What about multiple sequential zeros?



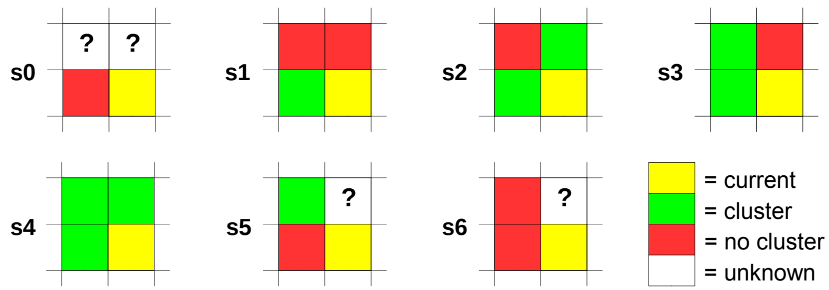
- This produces one last state:





Nearest-8 HK with FSM

Putting it all together...



Nearest-8 HK with FSM

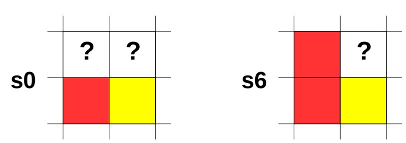
Details...

- Previous slide is missing a final state
  - In formal definition, a terminal symbol is specified, to be located after last cell
    - From any state, encountering this symbol leads to final state
  - Implementation does not include final state explicitly
    - Bounds checking used instead

Nearest-8 HK with FSM

More details...

- Row transitions
  - If matrix is padded on both sides with buffer columns of all zeros, FSM will reset to s_0 before proceeding to next row
  - In actual implementation, no buffer columns
    - Again, explicit bounds checking performed
    - At beginning of row, FSM reset to s_6
    - At end of row, last cell handled as special case



Nearest-8 HK with FSM

In action...

1	0	2	0	0	3	3	0
-1	0	-1	0	-1	-1	0	-1

1	1
2	1
3	2
4	0

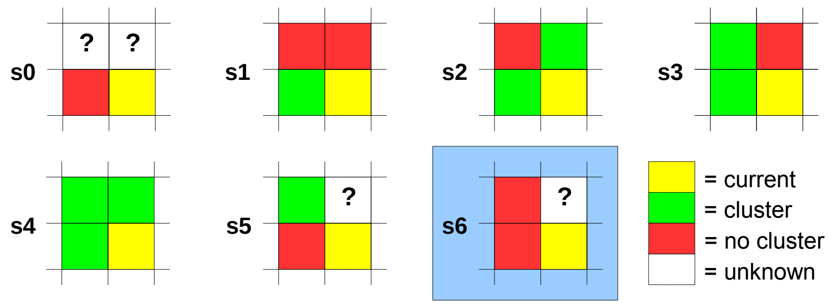
- Start with first row clustered as before

Nearest-8 HK with FSM

In action...

1	0	2	0	0	3	3	0
-1	0	-1	0	-1	-1	0	-1

1	1
2	1
3	2
4	0

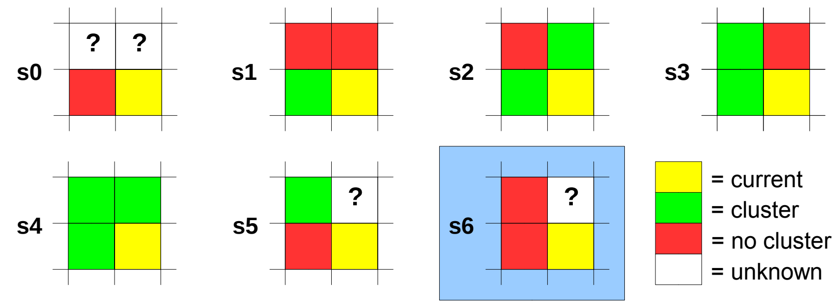


Nearest-8 HK with FSM

In action...

1	0	2	0	0	3	3	0
1	0	-1	0	-1	-1	0	-1

1	2
2	1
3	2
4	0

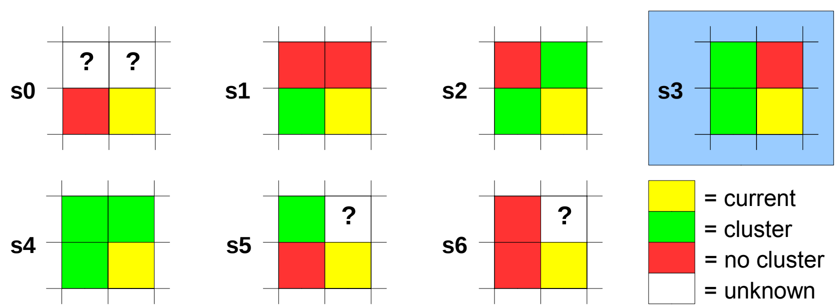


Nearest-8 HK with FSM

In action...

1	0	2	0	0	3	3	0
1	0	-1	0	-1	-1	0	-1

1	2
2	1
3	2
4	0

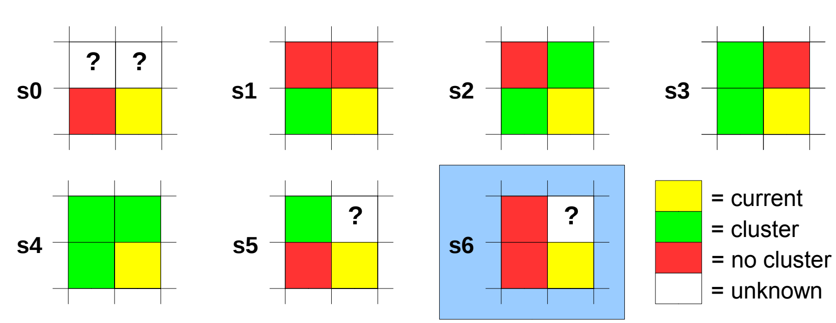


Nearest-8 HK with FSM

In action...

1	0	2	0	0	3	3	0
1	0	-1	0	-1	-1	0	-1

1	2
2	1
3	2
4	0

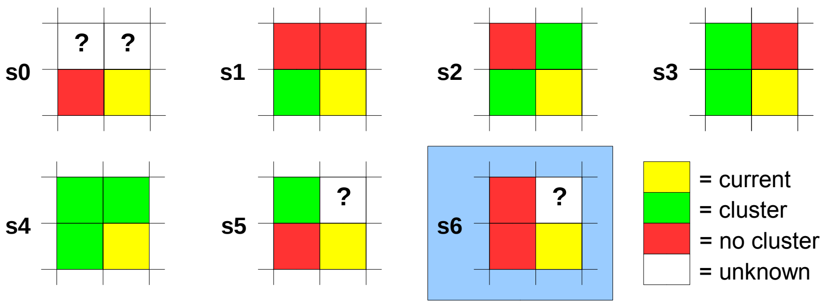


Nearest-8 HK with FSM

In action...

1	0	2	0	0	3	3	0
1	0	2	0	-1	-1	0	-1

1	2
2	2
3	2
4	0

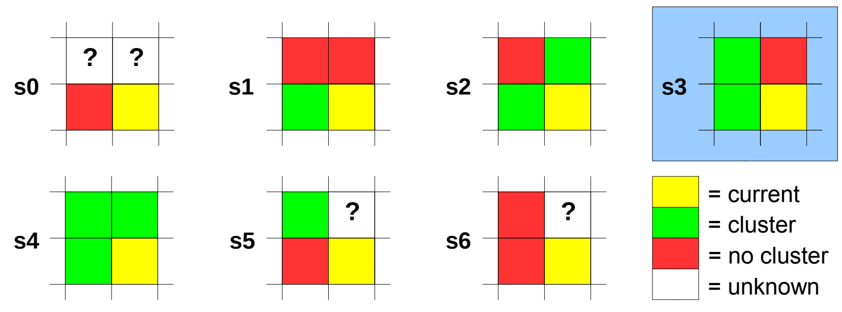


Nearest-8 HK with FSM

In action...

1	0	2	0	0	3	3	0
1	0	2	0	-1	-1	0	-1

1	2
2	1
3	2
4	0

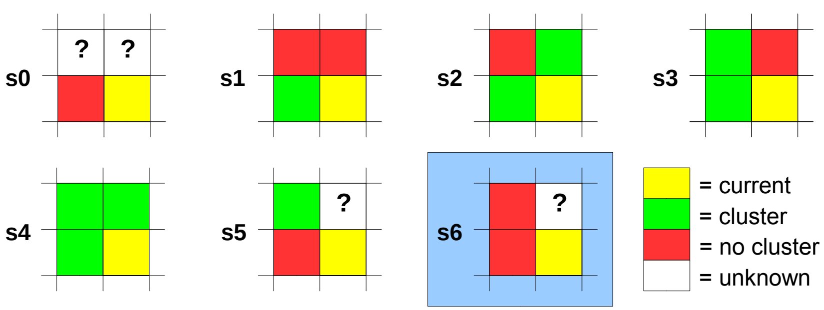


Nearest-8 HK with FSM

In action...

1	0	2	0	0	3	3	0
1	0	2	0	-1	-1	0	-1

1	2
2	1
3	2
4	0

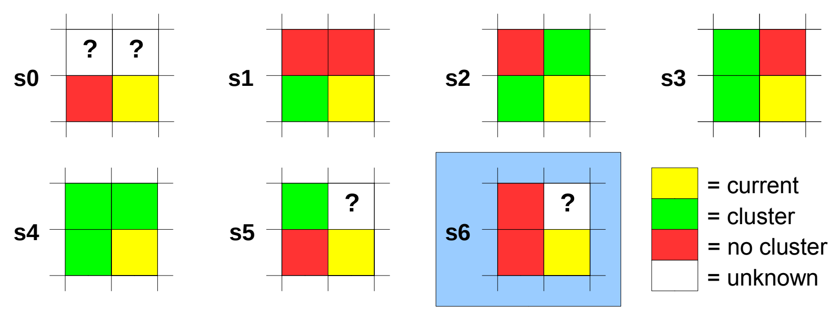


Nearest-8 HK with FSM

In action...

1	0	2	0	0	3	3	0
1	0	2	0	3	-1	0	-1

1	2
2	1
3	3
4	0

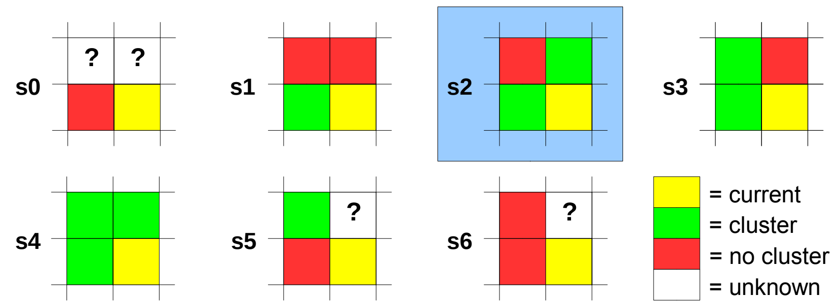


Nearest-8 HK with FSM

In action...

1	0	2	0	0	3	3	0
1	0	2	0	3	-1	0	-1

1	2
2	1
3	3
4	0

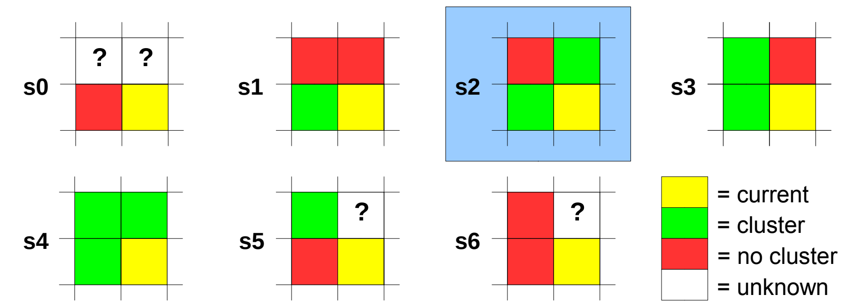


Nearest-8 HK with FSM

In action...

1	0	2	0	0	3	3	0
1	0	2	0	3	3	0	-1

1	2
2	1
3	4
4	0

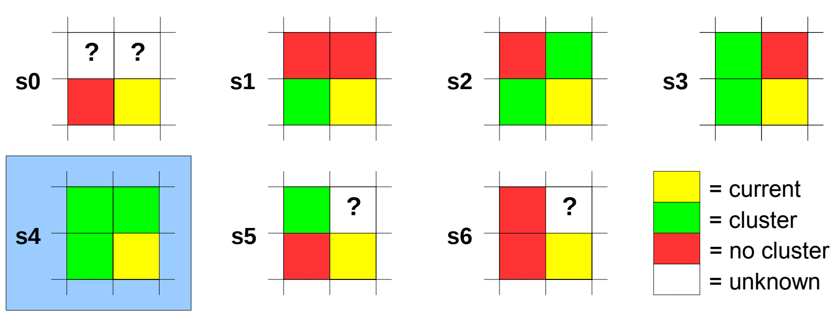


Nearest-8 HK with FSM

In action...

1	0	2	0	0	3	3	0
1	0	2	0	3	3	0	-1

1	2
2	1
3	4
4	0

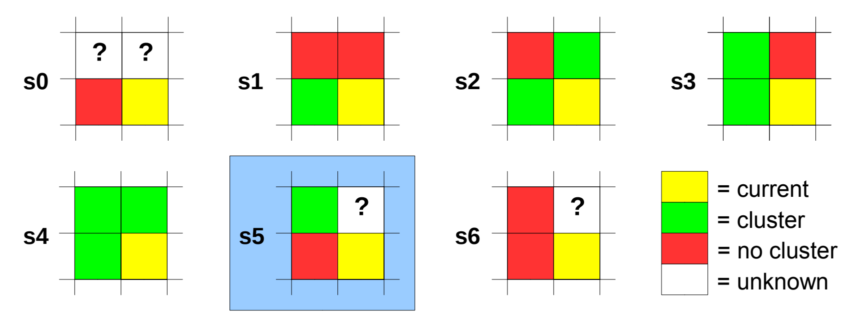


Nearest-8 HK with FSM

In action...

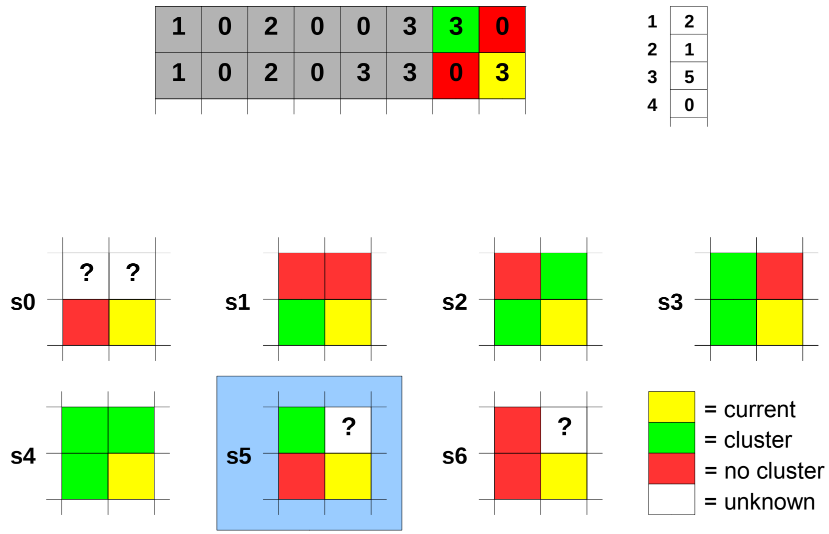
1	0	2	0	0	3	3	0
1	0	2	0	3	3	0	-1

1	2
2	1
3	4
4	0



Nearest-8 HK with FSM

In action...



Nearest-8 HK with FSM

Alternative Implementations

- Parallel computing
  - MPI used for process communication
  - Controller/worker design, round-robin job assignment
  - Matrix divided row-wise into *s* segments
  - csize also divided into *s* segments, with mutually exclusive cluster ID spaces
  - Results merged by controller node
  - Minimal speedup, mostly due to staggered I/O
  - May be useful for *much* larger data than used here

Nearest-8 HK with FSM

Alternative Implementations

- Concurrent FSMs
  - Identify multiple target classes in single pass
  - Each FSM maintains separate state
  - No longer in-place
  - Must maintain explicit state variables, rather than separate blocks of execution and implicit state

Workstation Performance

Methodology

- Tests performed on Linux workstation
  - 2.4 GHz Intel Xeon
  - 8 KB L1 cache
  - 512 KB L2 cache
- Timed over complete cluster analysis
  - First AND second pass (relabeling)
  - File I/O and data structure initialization not included
- Average time of 40 executions for each implementation and parameter set

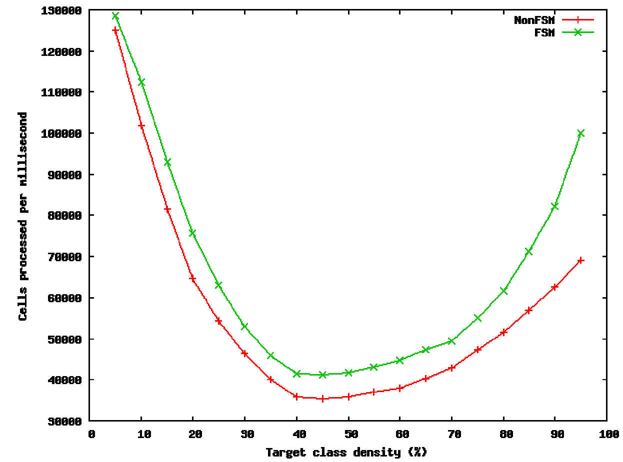
Workstation Performance

Test Data

- One set of 5000x5000 randomly generated binary matrices
  - Target class densities: { 0.05, 0.1, 0.15, ..., 0.95 }
- Three actual land cover maps
  - 2771x2814 Fort Benning, 15 classes
  - 4300x9891 Tennessee Valley, 21 classes
  - 400x500 Yellowstone, 6 classes

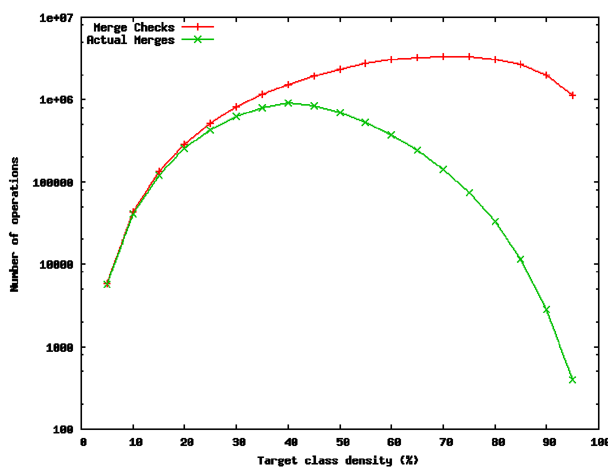
Workstation Performance

Random Data Results



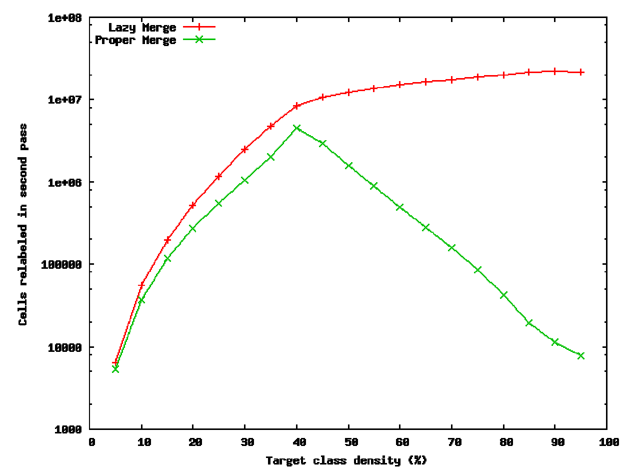
Workstation Performance

Random Data Results



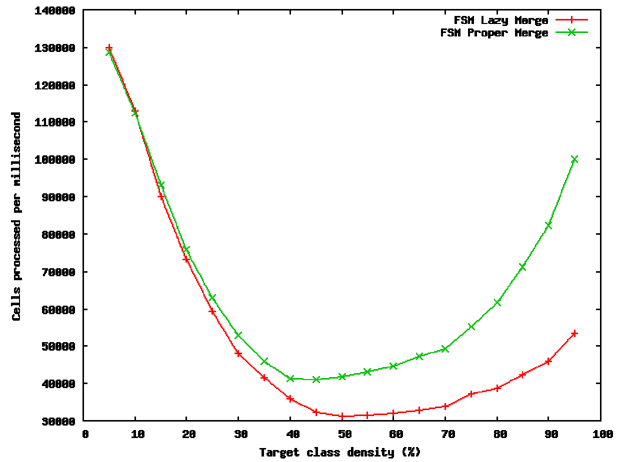
Workstation Performance

Random Data Results



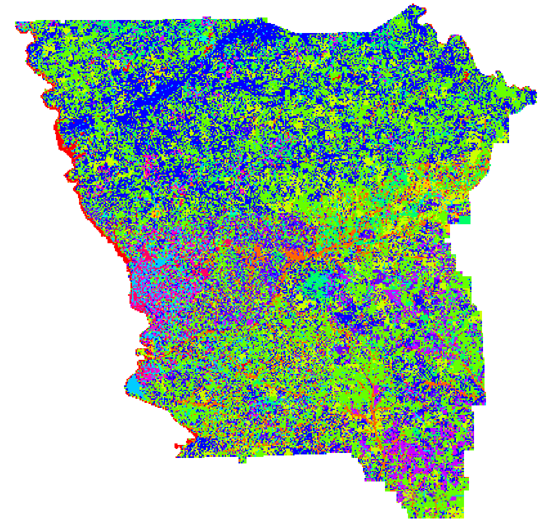
Workstation Performance

Random Data Results



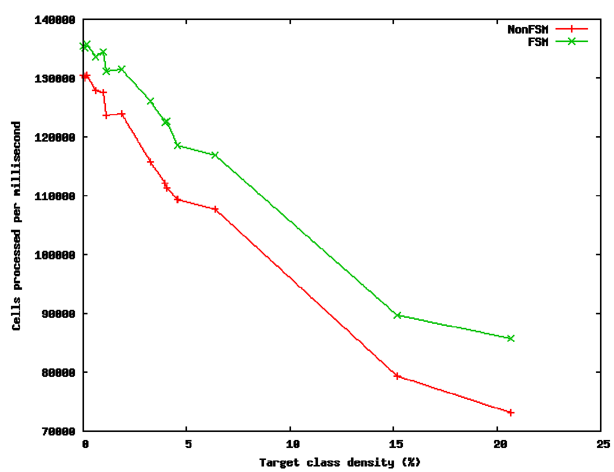
Workstation Performance

Fort Benning Data



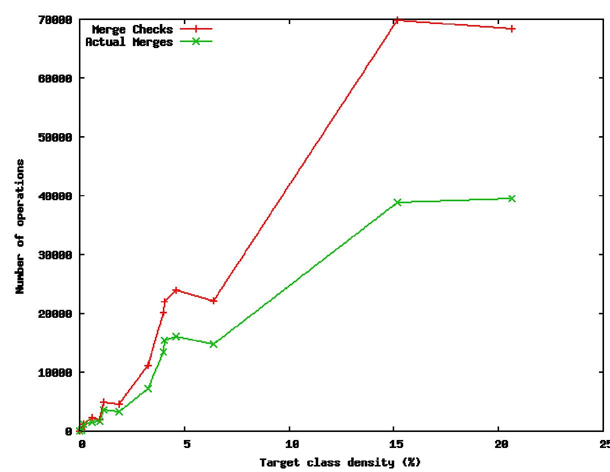
Workstation Performance

Fort Benning Data Results



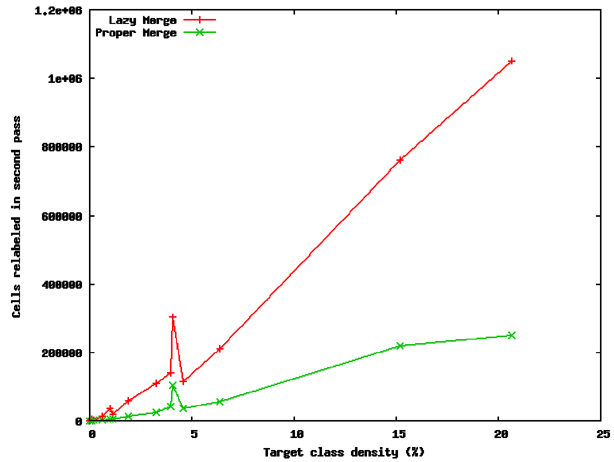
Workstation Performance

Fort Benning Data Results



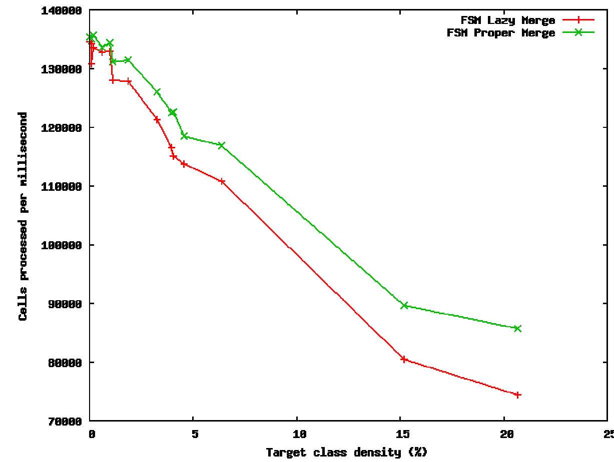
Workstation Performance

Fort Benning Data Results



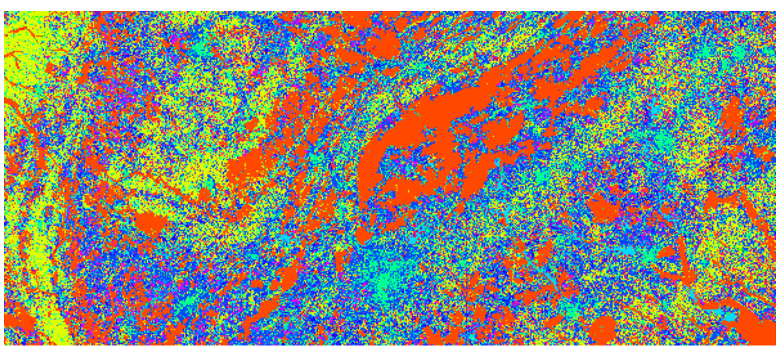
Workstation Performance

Fort Benning Data Results



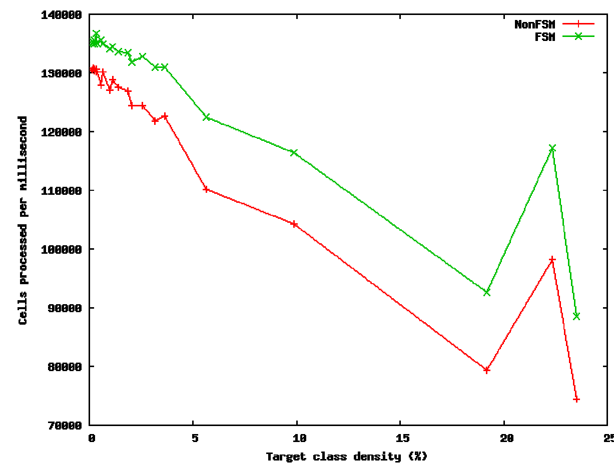
Workstation Performance

Tennessee Valley Data



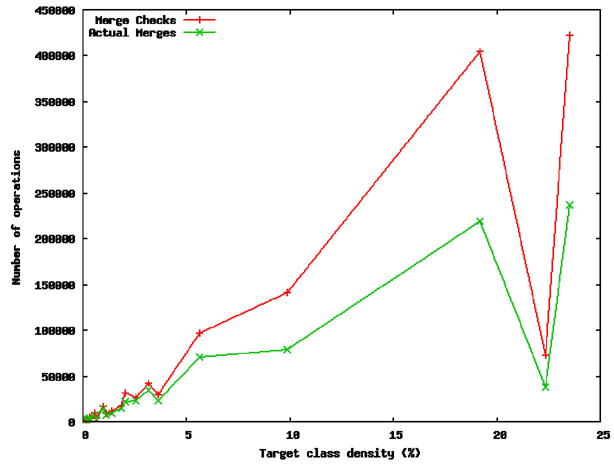
Workstation Performance

Tennessee Valley Data Results



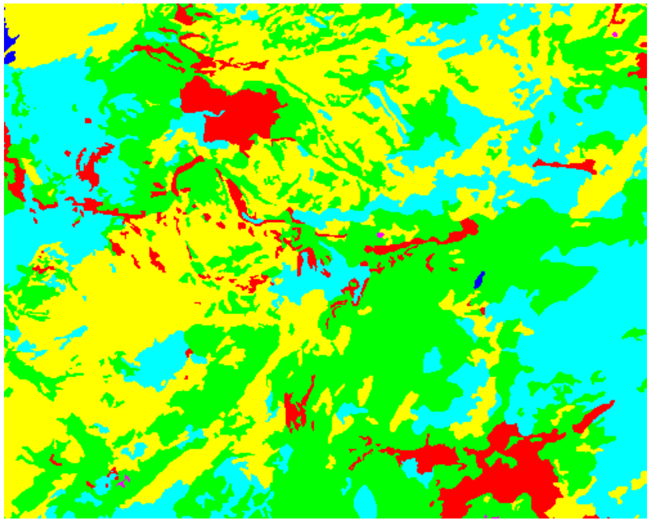
Workstation Performance

Tennessee Valley Data Results



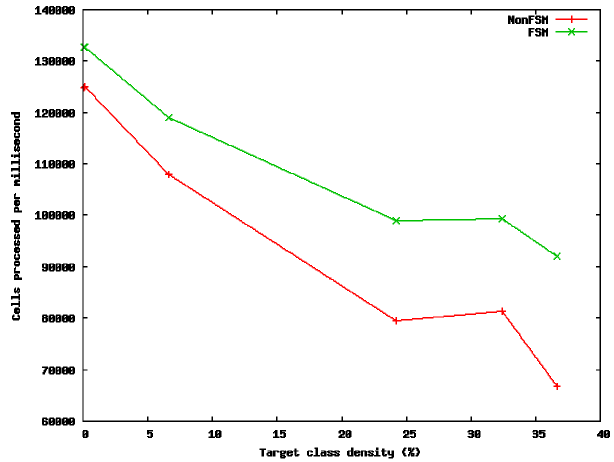
Workstation Performance

Yellowstone Data



Workstation Performance

Yellowstone Data Results



Workstation Performance

Conclusions

- FSM clearly outperforms non-FSM for both landscape and random data
  - Sparse clusters: non-FSM still competitive
  - Dense clusters: FSM advantage increases due to retaining knowledge of neighbor values more often
- Proper merging (using union by cluster size) is key to performance

Palm PDA Performance

Why a PDA?

- Perhaps FSM can shine in high-latency memory system
- Conceivable applications include...
  - Mobile computing for field researchers
  - Cluster analysis in low-powered embedded systems

Palm PDA Performance

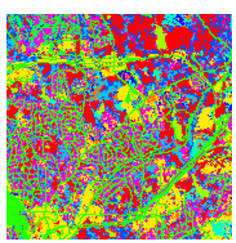
Methodology

- Tests performed on Palm IIIxe
  - 16 MHz Motorola Dragonball 68328EZ
  - 8MB RAM
  - No cache
- Only one run per implementation and parameter set
  - Single-threaded execution gives very little variation in run times (within 1/100 second observed)
- Very small datasets

Palm PDA Performance

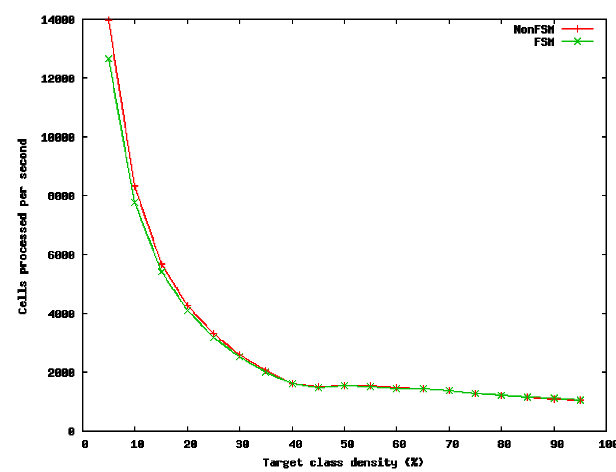
Test Data

- One set of 150x150 randomly generated binary matrices
  - Target class densities: { 0.05, 0.1, 0.15, ..., 0.95 }
- 175x175 segment of Fort Benning map
  - 13 target classes



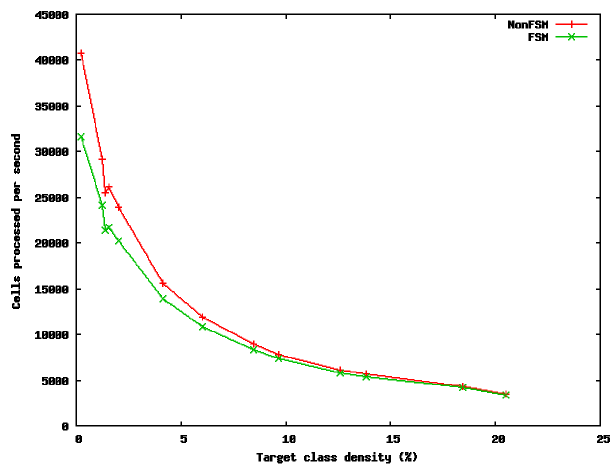
Palm PDA Performance

Random Data Results



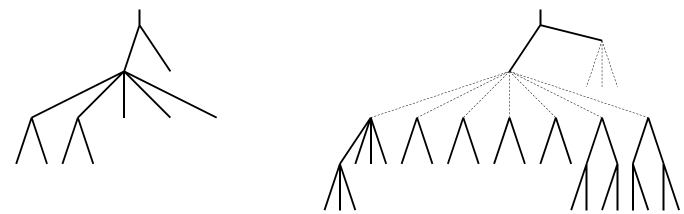
Palm PDA Performance

Fort Benning Data Results



Palm PDA Performance

Branching in FSM vs. Non-FSM



non-FSM

FSM
(dashed lines indicate
state-based branching)

Palm PDA Performance

Conclusions

- Non-FSM implementation faster in all cases
  - FSM more competitive with higher target class densities
- Why is the FSM slower?
  - Ironically, lack of cache
  - Also, reduced program locality and execution branching
  - Adding as little as 1-2 KB of cache can reduce Palm's effective memory access time by 50% (Carroll, et al.)

In Closing

Possible Future Work

- Extension to three (or higher?) dimensions
  - Higher dimensions => more neighbors => many more states
    - Automated FSM construction would ease burden, allow non-programmers to define custom neighborhood rules
  - If effects of complex control logic/branching can be mitigated, then FSM savings should be great
- FSM adaptation for different data ordering (e.g. Z- or Morton-order)
- Implement FSM HK in hardware (FPGAs, etc.)